The Volume-Return Relationship under Asymmetry of Information and Short Sales Prohibitions

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In the presence of asymmetric information, stock trading is particularly affected when market participants are prohibited from short selling. Although insiders privy to negative information may not exploit this information by selling if they do not own the stock, the market maker can deduce the occurrence of bad news by observing the trading patterns. Prices are shown to rise on high volume after good news and fall on low volume given bad news; furthermore, the speed of adjustment is greater on bad news, that is, the price falls more quickly to a lower equilibrium price than it rises to the price corresponding to good news. The second result depends upon parameters determining the structure of the market in terms of the types of participants (informed or uninformed) and their relative holdings of the stock. The volume anomaly is confirmed by an empirical study of trading on the Stock Exchange of Thailand, where short sales are prohibited. The results show that high trading volume in a given day is a strong indicator of high expected returns in the next period for almost all the stocks in our sample.

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Inside information and restrictions on trading opportunities are generally considered to be detrimental to market efficiency. Market makers adjust their prices and spreads based on observation of the orders they receive from unidentified traders potentially having superior information. Both the number of buy and sell orders and the volume of trading are partially revealing of the true nature of existing information. If short selling is prohibited, then informed traders aware of bad news will have sell orders restricted by their holdings, implying that a low volume of orders potentially reveals the occurrence of bad news. Hence the observation of no trades may be indicative of the presence of informed would be sellers. The market maker's quoting is based an estimate of the distribution of the true price, which follows a diagnosis of order flow from unidentified informed and uninformed traders; this process benefits from awareness of the ratio of informed traders, and of the stock holdings of informed and uninformed investors.

This paper modifies and extends the Easley and O'Hara [1992a] model of securities trading under asymmetric information by the incorporation of short sales prohibitions. Contrary to their linkage of price adjustment to buy and sell orders alone, we show that the observation of no trades also leads to revision of the price distribution. When short sales are prohibited, the asymmetric interpretation given to no trades causes an asymmetric volume reaction to good and bad news. We also demonstrate that under mild conditions on the market structure, there is a positive intertemporal correlation between volume and return, with volume leading. Additional microstructure conditions enable a faster price adjustment to bad news than to good news. In effect, we find the effect of a short sale prohibition to be beneficial to efficiency, in contrast to prior results. More significantly, we show that information asymmetry and the prohibition of short sales together provide an opportunity to gain information about the intertemporal pattern of stock returns, thereby offering a possible justification for technical analysis.

Our conclusions are supported by a study of recent trading on the Stock Exchange of Thailand, where short selling is prohibited. This study confirms the predicted volume anomaly by demonstrating high returns on high volume stocks and the correlation between volume and return in the next period. This was supported by the finding of increasing returns by volume from volume-decile portfolios, and by positive correlations between volumes and next-day returns for sixty out of sixty individual companies studied. This result is reinforced by a simulation of the returns over the test period from a portfolio constructed from a high volume filter. The recorded return was 622% over a two-year period; however, the reference portfolio also had a 297% return from buy-and-hold or 211% if re-balanced for equal weighting at this time.

Market microstructure under asymmetric information has been the focus of numerous studies. Kyle [1985] analysed order flow in continuous time to show that information is gradually incorporated into prices over time as informed traders could generate more profits from continuous trading than from strategic order entries. Foster and Viswanathan [1990] modified this approach and found a relationship between size of orders and the activity of a monopolist insider, with the result that the uninformed could gain information from observing the trade history. Glosten and Milgrom [1985] created a sequential equilibrium model with prices equal to the market maker's conditional expectation of asset value based on trade flow, finding volume not to be increasing in the variance in prices. Diamond and Verrecchia [1987] extended Glosten and Milgrom's model to include three types of short sale constraints. They examined whether market short sale constraints affect the trading propensity, thereby causing asymmetries in the speed of price adjustment to good and bad news. They concluded that a short-sale prohibition reduced the speed of price adjustment to inside information.

Easley and O'Hara [1987] used a modified, discrete time Glosten and Milgrom approach to include different trade sizes and information uncertainty. The market maker attempts to determine both the existence and nature of new information. Easley and O'Hara [1992a] (henceforth EO) focused on the trade process instead of the usual price process, finding that event uncertainty provides an informational role for trading volume gained directly from the properties of the underlying information structure, with time and volume becoming endogenous variables. In contrast to the auction-type market equilibrium mechanisms of other models, the EO approach focuses on the trade process, by postulating exogenous random arrivals for both information and traders (informed and uninformed). Price quotes are offered by a risk-neutral market maker, who engages in a Bayesian updating of the quotes as a function of each successive trading order. The arrival processes of both information and traders can be either discrete, as in Easley, Kiefer and O'Hara [1995], or continuous, as in Easley, Kiefer, O'Hara, and Paperman [1996]. By opening up the possibility of a 'no trade' event to all agents, the model predicts the positive correlation between price observation and trading volume. Brennan and Subrahmanyam [1995a] found that privately informed investors create significant illiquidity costs for uninformed investors while Foster and Viswanathan [1995] found a model of speculative trading to be partially consistent with the asymmetric volume-volatility relationship, as evidenced by intra-day transaction data for an individual firm during 1988.

The basic structure of asymmetric information models consists of at least two types of traders, informed and uninformed; transactions occur in a pure exchange economy in which the informed traders possess more precise knowledge than the uninformed about future values of economic variables affecting their current choices. Of particular interest are conditions leading to the full revelation of the knowledge possessed by informed traders. Early asymmetric information studies focused on the well-known paradox, first presented by Grossman [1976, 1978], and Grossman and Stiglitz [1980]: If prices are fully revealing and information acquisition is costly then there is no incentive to invest in acquiring information. The solution to this paradox comes from independent sources of randomness, where private information exists but is not fully revealing by equilibrium prices and volumes.

Asymmetric information has also been suggested as a justification for technical analysis which is refuted by the efficient markets hypothesis. For instance, Brown and Jennings [1989], Grundy and McNichols [1989], and Kim and Verrecchia [1991a,b] have investigated equilibrium market pricing under asymmetric information in sequential trading in two-period models that follow the pioneering formulation of Kyle (1985, 1989); their approach has been extended by

Vives [1995] to any number of periods. These authors find that observing a sequence of trades (price and volume) reveals information to market participants, as claimed by technical analysts.

The volume-return relationship has received increasing attention of late, with Karpoff [1987] giving an earlier survey of the field. Campbell, Grossman and Wang [1993] linked volume to serial correlation in price, while Lo and Wang [2000a,b,c] examined the volume relationship to portfolio returns. Llorente, Michaely, Saar and Wang [2001] (LMSW) investigated individual stock returns and found runs and reversals were dependent on the motivation of traders (risk-sharing versus speculation); in fact, they concluded that return continuation occurs on high volume days when high information is present (and conversely for low information), which supports our findings. Their results, however, were disputed by Ayogdu [2001], based on examination of the net order flow. Liquidity aspects motivated the investigation of the time series properties of stocks by Chordia, Roll and Subrahmanyam [2001, 2002], Huberman and Halka [1999] and Hasbrouck and Seppi [1999], who all determined that liquidity and trading activity were variable. Particularly relevant to our study are the findings of positive correlation between depth and trading activity prior to major announcements, and the link of stock inventory and asymmetric information to trading activity.

Most studies provide empirical evidence as to market phenomena, whereas both the LMSW approach and ours offer theoretical foundations for the observed results. The similar conclusions of LMSW, however, derive from some fundamental differences in approach from ours. Our perspective from the market maker's position contrasts with their general equilibrium view of risk averse utility maximisation by investors. The learning process for the risk-neutral market maker in estimating price differs from the use of volume and price by investors to foresee price evolution. Furthermore, although both studies focus on individual stock prices and volumes, LMSW examine both firm-specific and market news and their effects, while we restrict information to firm-specific events. Their utility maximization requires assumption of CARA functions and i.i.d. stock behaviour in order to derive a closed form solution closed form solution.

They defend their identification of a similar turnover anomaly to ours by appeal to NYSE and AMEX data from 1992-98; this setting clearly does *not* involve short sale prohibition, but *does* involve specialists who are not included in their model

The primary interest of the Diamond and Verrechia study was the effect of short sales restrictions on stock market efficiency, measured by the speed of convergence of the Bayesian prices to their "correct" values[1]. Their finding that short sales prohibitions reduce the speed of convergence implies that they contribute negatively to market efficiency. This is contrary to our result using the EO framework when short sales prohibitions are introduced. Even though the increased predictability of prices is contrary to efficiency, we conclude that a short sales prohibition contributes to *faster* convergence of prices to their correct values. Bhattacharya and Daouk [2001] provide evidence that emerging markets such as Thailand have much lower probabilities of prosecution for insider trading (less than 25% chance versus over 80% for the U.S.); this motivates using Thailand as a test ground for our hypotheses.

In the next section we describe the general formulation and notation of our model. The subsequent section presents the main result concerning the effect of the absence of short sales, namely that the short selling prohibition leads to reduced volume on selling associated with bad news in comparison to increased volume on buying associated with good news. Following that, we shall derive the theoretical results concerning speed of convergence of the estimated price distribution to that known only to informed investors; to that end, we present the statistical concept of entropy for measuring the convergence of distributions. We explore how the lower volume associated with bad news adjusts the price to the decreased equilibrium value; the result is shown to be a speedy adjustment on low volume, that is a relatively efficient price revision compared to the good news case. This conclusion depends on some of the parameters of the model, but the indicated value ranges are reasonable and have logically consistent limiting cases. In the following section, we present an empirical study based on trading on the Stock Exchange of Thailand (SET) in 1998 and

[1] This was also investigated in Easley and O'Hara (1992a), but without short sales constraints.

1999. This study confirms the volume anomaly, as high volume stocks exhibit high returns in the following period with significant correlation. To illustrate the predictive value, a simulation is conducted of the returns from a high-volume portfolio over the test period.

I. INTRODUCTION TO THE MODEL AND PARAMETERS

We assume a single risky asset traded without transaction costs in a pure and competitive dealership market in a single period, divided into equal-length trading intervals. The market has three types of risk-neutral participants: informed traders or insiders, uninformed liquidity traders, and market makers. At the beginning of each trading interval (t=0) informed traders observe privately a signal Ψ perfectly correlated with the true value of the risky asset. Let $\alpha \in (0,1)$ denote the probability of an informational event occurring. The informed trader will use the private information to buy if the asset is underpriced and sell all owned shares if overpriced. The asset is overpriced (underpriced) if its asking price is more (less) than the trader's conditional expectation of its liquidating value given the signal Ψ. An informed trader will not trade in the absence of information or if lacking the stock to sell.

We also have liquidity traders, who are price takers and uninformed about Ψ. They trade the risky asset for exogenous non-informational reasons or portfolio considerations (consumption needs, tax planning, etc.), selling and buying randomly, with respective probabilities γ and $1-\gamma$. Finally, market makers are uninformed about Ψ, or about the future value of the risky asset[2]. Each market maker sets prices at which he will be ready to buy or to sell with any traders for at most one unit of the traded asset at any time, based on observation of the order flow from traders unidentifiable by him.

Each trading day is divided into n equal discrete time subdivisions, denoted by t=1,...,n. The signal Ψ is observed by the informed traders at some time prior to t=0.The trading interval is

2 For simplicity, we refer to the actions of a single market maker, but the assumption of competitive behaviour requires the existence of at least one potential competitor.

designed to be sufficiently small that at most one trade may take place, with the result that no trade may also occur. Hence, at each subdivision we observe one out of three mutually exclusive events, drawn from the set {B,S,N}, for buy, sell and no trade.

In accordance with EO, the model assumes that the market maker chooses randomly either an informed or a liquidity trader from the population of all traders, with respective probabilities μ and $1-\mu$, where $\mu \in (0,1)$[3]. Both types of traders choose their strategies from the set {B,S,N} when trading with the market maker. Whenever the chosen agent does not own the asset, a desired sell order is replaced by a no trade. h_I and h_U denote the respective probabilities that informed and uninformed traders own the asset, where $h_j \in (0,1)$ for j=I,U, so that an N outcome occurs with probability $1-h_j$ if the selected trader wishes to sell but does not own the stock. By contrast, an N outcome cannot result from any trader wishing to buy. This asymmetry between buys and sells causes the results of this model to differ from those of EO.

Neither market maker nor uninformed traders knows if the informational event Ψ has occurred or, if it has, whether it is "good news" or "bad news". All agents know the structure of the economy. Figure 1 summarizes the tree diagram of the trading process induced by the model.

(**Figure 1 about here**)

In the absence of an informational event, the eventual value of the risky asset is a universally known random value V per share, with positive mean and variance. When an informational event occurs, observed only by the informed traders, the signal Ψ is given, which is low with prior probability δ or high with probability $1-\delta$, where $\delta \in (0,1)$. We characterise the signal then by $\Psi=\{L,H,0\}$, where L (H) denotes that the risky asset has low (high) value and $\Psi=0$ denotes the event of no information. Let also $V_L \equiv E\{V|\Psi=L\}$ and $V_H \equiv E\{V|\Psi=H\}$ denote the

3 A summary of the extensive notation is given in Table A1, in the appendix.

conditional expectations given that the indicated informational event has occurred. By contrast, for Ψ=0 the unconditional expected value of the asset is given by $V^*=\delta V_L+(1-\delta) V_H$; note that $V_H>V^*>V_L$.

To summarise the trading strategies of both types of traders when an informational event occurs, the informed trader, selected with probability μ, chooses strategy B if the stock is underpriced and S if it is overpriced, the latter only with probability h_I. By contrast, the strategy N is chosen if the desired strategy is S but he owns no stock, an event with probability $1-h_I$. As for the liquidity trader, selected with probability $1-\mu$, the strategies S and B are chosen randomly with probabilities γ and $1-\gamma$, except that the outcome N may result with probability $1-h_U$ whenever S is chosen. Finally, on days when an informational event does not occur there are no informed traders present, while liquidity traders' behaviour is unchanged.

The comparison of the tree in Figure 1 with the corresponding figure of EO shows two main differences, only one of which is relevant to the results of this paper. In the earlier study the possibility of an N strategy arises, with probability $1-\varepsilon$, each time a liquidity trader is selected, irrespective of whether the alternative is B or S. In our case, by contrast, an N strategy occurs in both informed and uninformed traders (albeit with different probabilities), but only in conjunction with an S strategy. Hence, the EO model corresponds observationally to ours[4] when h_I=1 and $h_U\in(0,1)$.

Since the focus of this paper is on the price adjustment process, we provide here only a brief outline of the sequential revision of probabilities and resulting quote formation, which parallel the EO development. At t=0 the market maker's <u>ex ante</u> probabilities are δ and $1-\delta$ for V_L and V_H respectively. These probabilities are revised after observing the trades submitted by the

[4] An alternative formulation would be to include informed traders entering the market with an N strategy when there is no signal. Also, our model can easily incorporate an N strategy for liquidity traders under all circumstances, as long as the probability of N being chosen is higher in association with an S than with a B alternative. What drives our results is the asymmetry between buys and sells on the part of all traders.

unidentified informed or liquidity traders. Given the first trade Q, therefore, the ex post probability $\delta(Q)$ is defined by:

$$\delta(Q) = \Pr \{V=V_L \mid Q\}$$

This probability is then calculated as:

$$\delta(Q) = \Pr \{V=V_L \mid \Psi=L\} \bullet \Pr\{\Psi=L \mid Q\} + \Pr\{V=V_L \mid \Psi=0\} \bullet \Pr\{\Psi=0 \mid Q\}$$

$$= 1 \bullet \Pr \{\Psi=L \mid Q\} + \delta \bullet \Pr\{\Psi=0 \mid Q\} \qquad (1)$$

since $\Pr\{V=V_L \mid \Psi=H\}=0$. The probability $1-\delta(Q)=\Pr\{V=V_H \mid Q\}$ is found similarly.

The ex post probabilities $\Pr \{\Psi=X \mid Q\}$ of a particular signal X given a trade outcome Q are calculated by Bayesian formulas. From the tree diagram in Figure 1, which gives the individual probabilities along each path and the joint probabilities along each terminal node, we get the following revisions of expectations if a sale occurs:

$$\Pr \{\Psi=L \mid S\} = \Pr \{\Psi=L\} \bullet \Pr\{S \mid \Psi=L\}/\Pr\{S\}$$

$$= [\alpha\delta\{\mu h_i+\gamma(1-\mu)h_U\}] / [\alpha\delta h_I+\gamma h_U(1-\alpha\mu)] \qquad (2)$$

$$\Pr \{\Psi=H \mid S\} = \alpha(1-\delta)(1-\mu)\gamma h_U / [\alpha\delta\mu h_I+\gamma h_U(1-\alpha\mu)] \qquad (3)$$

$$\Pr \{\Psi= 0 \mid S\} = (1-\alpha)\gamma h_U / [\alpha\delta\mu h_I+\gamma h_U(1-\alpha\mu)] \qquad (4)$$

Substituting now (2)-(4) into (1), we get the revised probability of a low signal given a sale:

$$\delta(Q=S) = \delta\{\alpha\mu h_I + (1-\alpha)\gamma h_U\} / [\delta\alpha\mu h_I + (1-\alpha\mu)\gamma h_U)] \qquad (5)$$

It can be easily seen that if the first trade is a sale then $\delta(Q=S)>\delta$, implying that the probability of a low signal is revised upwards. Similarly, it can be shown that if the first trade is a purchase then $\delta(Q=B)<\delta$, while $\delta(Q=N)=\delta$.

Briefly the process of quote revisions is as follows. Suppose the first trade is a sale; the initial bid price would then be set equal to:

$$E [V \mid S] = \Pr\{V=V_L \mid S\}\bullet V_L+ \Pr\{V=V_H \mid S\}\bullet V_H= \delta(Q=S) \bullet V_L+[1-\delta(Q=S)] \bullet V_H$$

where the revised probability $\delta(Q=S)$ is given by (5). Similarly, the initial ask price would be set if the first trade that arrives is a purchase, with the value of

$$E[V|B] = \delta(Q=B) \bullet V_L + [1-\delta(Q=B)] \bullet V_H.$$

II. EVOLUTION OF THE PRICE PROCESS

The process used by the market maker to establish a semi-strong equilibrium price based on a single trade indicates how prices are set following a sequence of orders received during the trading day. In fact, a period with a no trade occurrence conveys useful information, due to the short-sale prohibition. In this section, we present a number of interim results that characterize the pattern of trades as the market maker reassesses his belief about the existence of a signal. These culminate in the correlation of volume of trading with the price direction. Our conclusion about the short sale prohibition effect on the price-volume relationship is at odds with He and Wang [1995], who found that private information not only generates increased trading in the current period, but also leads to possible trading in a future period, both on good news and bad news.

Revision of Beliefs

With an asymmetric information structure, the evolution of the order flow is endogenous to the market maker's problem of protecting himself from trading based on informed information, and of discovering the nature of that information. Hence, trade itself conveys information on the underlying asset to the market maker. The issue is then how the market maker learns from the trading activities and how this will affect the evolution of the price process. Let Q_t denote the outcome of a trade event at time t (i.e. $Q_t \in [B,S,N]$), and let Q^{t-1} be a vector of past sequential trade information set at time t-1 (i.e. $Q^{t-1} = [Q_1, Q_2 ... Q_{t-1}]$). The stochastic process of prices determines the process of the market maker's beliefs, indicated by either his revised conditional probability that the price is V_t, or his revised conditional expectation for the value. Let $\rho_{\psi, t}$ denote the conditional probability that the signal $\psi \in [H,L,0]$ has occurred as of the beginning of period t, given the history of previous trades Q^{t-1}. Then let $\rho^{Q}_{\psi, t+1}$ denote for the following period the conditional probability of that signal given the past trading history Q^{t-1} followed by the most recent trade Q_t, permitting a recursive means of describing revision. The first two results parallel the EO development.

Proposition 1: If there is *no trade* at time t, then

a) The sign of the difference between the revised probability of a no information event and its previous value ($\rho^{N}_{0,t+1} - \rho_{0,t}$) depends on the prior period's probability and the parameters h_I, h_U, γ and μ.

b) The sign of the difference between the revised probability of a low signal and its previous value ($\rho^{N}_{L,t+1} - \rho_{L,t}$) depends on the prior period's probability and the parameters h_I, h_U, γ and μ.

c) The difference between the revised probability of a high signal and its previous value ($\rho^{N}_{H,t+1} - \rho_{H,t}$) is negative.

Proof : See the appendix.

As found by EO in their first proposition, here also the market maker learns from both a no trade situation and from an actual trade. In contrast to EO, however, the information of no trade concerning the high signal is more conclusive than for the no or low signal case. The occurrence of no trade decreases the belief of the market maker that a high signal has been given, but his beliefs about the other two signals are very dependent on the parameters describing the market composition.

The absence of trading by an informed trader occurs only due to either no news or no stock possession. If all the informed traders own shares or if short sales are allowed (i.e. $h_I = 1$), the difference in a) will be positive; hence, if all informed traders own the stock, then no trade may be interpreted as no news as in EO. As h_I decreases below 1, this interpretation becomes less plausible, since no trade also may imply no stock ownership. At some value $h_I > h_U$, the inequality in a) can easily be reversed for various values of $\rho_{o,t}$, $\rho_{L,t}$ and γ. At $h_I = h_U$, the inequality is reversed for all other parameter values[5]. (See the details in the appendix.) In case b), for $h_I = 1$, the difference will be negative; if all informed traders own the stock and receive bad news, each will sell. Hence, no trade will decrease the likelihood of bad news in the market maker's estimate. Again, as h_I decreases, the inequality can be reversed; at $h_I = h_U$, the inequality is reversed for all parameter values. When both h_I and h_U are 1, the model reduces to the EO findings, being equivalent to allowing short sales; it is the possibility of informed traders having no stock (i.e. h_I <1) that leads to the opposite of their results.

Corollary: For $0<h_I<1$ and all other parameter values, the market maker's assessment of the relative probabilities of low to high signals:

a) increases if there is no trade, i.e. $[\rho^N_{L,t+1} / \rho^N_{H,t+1}] > [\rho_{L,t} / \rho_{H,t}]$

b) increases if there is a sale, i.e. $[\rho^S_{L,t+1} / \rho^S_{H,t+1}] > [\rho_{L,t} / \rho_{H,t}]$

c) decreases if there is a buy, i.e. $[\rho^B_{L,t+1} / \rho^B_{H,t+1}] < [\rho_{L,t} / \rho_{H,t}]$

Proof : See the appendix.

Similarly to EO, the model predicts that no trade conveys information to the market maker. In contrast, while EO find that no trade indicates only the existence of the signal, this model infers both the existence and the nature of the signal. In other words, the inequality a) of the corollary informs the market maker that the relative probability of a low to a high signal increases after no trade is observed, whereas EO determine the probability as unchanged. One infers that given a 'no trade' event, the stock is more likely to move down than up under a short sale constraint.

Volume of Trading

Since at most one unit of the asset can be traded in each trading period, the time interval itself can serve as a counting reference. By construction, the total trading time is equal to the

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5 The condition $h_I = h_U$ has implications for various results in the paper.

summation of all trade events. Let $t \equiv$ total number of time periods within which trading events have occurred, $\beta_t \equiv$ total number of purchases within t, $s_t \equiv$ total number of sales within t, and $n_t = t - (\beta_t + s_t)$, the total number of no trade periods within t; then $v_t \equiv \beta_t + s_t$ denotes the total trading volume up to time t. We observe that the random variables β_t, s_t and n_t follow Markovian stochastic processes, as the arrivals of trades occur randomly and independently in each subperiod; this also implies that the distribution of n_t is binomial, depending only on the total number of buyers and sellers and not on the order of their arrival. The following two related results are direct consequences of the short sales prohibition.

Proposition 2: If not all informed traders own the risky asset ($0 < h_I < 1$), then

$$E[v_t | \Psi=H] > E[v_t | \Psi=L] \text{ and } E[v_t | \Psi=H] > E[v_t | \Psi=0].$$

Furthermore, $E[v_t | \Psi=0] > E[v_t | \Psi=L]$ if $\gamma(1-h_U) < (1-h_I)$.

Proof: By Figure 1, the probability of no trade within a single interval is equal to:

$\theta_L \equiv [\mu(1-h_I)+(1-\mu)\gamma(1-h_U)]$, when a low signal has occurred,

$\theta_H \equiv (1-\mu)\gamma(1-h_U)$, when there is a high signal, and

$\theta_0 \equiv \gamma(1-h_U)$ when there is no signal.

These quantities are also the parameters of the binomial distribution of the number n_t of no trades in a number t of intervals given the occurrence of the corresponding signal. Hence,

$$E[n_t | \Psi=H] = t(1-\mu)\gamma(1-h_U),$$

$$E[n_t | \Psi=L] = t[\mu(1-h_I)+(1-\mu)\gamma(1-h_U)],$$

$$E[n_t | \Psi=0] = t\gamma(1-h_u),$$

and it is immediately clear that $E[n_t | \Psi=H]$ is less than the other two. Then for $0<h_I<1$, $0<h_U<1$ and $0<\gamma<1$,

$$[\mu(1-h_I)+(1-\mu)\gamma(1-h_U)] > [\gamma(1-h_U)] \text{ if } \gamma(1-h_U) < (1-h_I).$$

The proposition follows immediately, since for any event X we have $E[v_t | \Psi=X] = t - E[n_t | \Psi=X]$. **QED**.

Note the interpretation of the condition that $1-h_I$ be greater than $(1-h_U)\gamma$, implying that the expected volume in the lower signal is less than in the no signal case. It states that the propensity of an informed trader to short sell is greater than that of an uninformed trader. Short selling by uninformed investors occurs only if they wish to sell and own the stock, which is described by $(1-h_U)\gamma$; for informed investors, an order to sell is not probabilistic but determined by news, and hence occurs if they own the stock. In this case, volume is monotonically associated with the quality of the signal; however, if this condition does not hold, then high volume is still associated with good news, but it is not clear whether low volume indicates a low or no signal. The information conveyed by the volume-signal relationship is captured by the principal result of this section.[6]

Theorem 1: There exists a positive and intertemporal relationship between trading volume and stock return, provided not all informed traders own the risky asset ($h_I<1$) and the probability of a low signal, δ, is sufficiently low, i.e.,

$$E[V_{t+1} | v_t = j] \text{ is increasing in } j$$

$$\text{for } \delta < [1+(\theta_L/\theta_H)^{(t-j)/2}((1-\theta_L)/(1-\theta_H))^{t/2}]^{-1} \quad \text{where } \theta_\psi = \Pr\{N | \psi\},\ \psi=L,H.$$

Proof: We know that:

$$E[V_{t+1} | v_t=j] = V_H \bullet \Pr\{\Psi=H | v_t=j\}+V_L \bullet \Pr\{\Psi=L | v_t=j\}+V \bullet \Pr\{\Psi=0 | v_t=j\}, \tag{6}$$

where $V^*=\delta V_L+(1-\delta)V_H$. The conditional probabilities in this expression are found from the relation $\Pr\{\Psi=X | v_t=j\} = \Pr\{v_t=j | \Psi=X\}\bullet\Pr\{\Psi=X\}/\Pr\{v_t=j\}$, where $X=\{L,H,0\}$, $\Pr\{v_t=j\} = \Sigma_X \Pr\{v_t=j | \Psi=X\}\bullet\Pr\{\Psi=X\}$, and the probabilities of $\{L,H,0\}$ are $\{\alpha\delta,\alpha(1-\delta),(1-\alpha)\}$, respectively. As for the probabilities $\Pr\{\Psi=X | v_t=j\}$, these are binomial with parameters θ_X. Hence, we have:

$$\Pr\{\Psi=H | v_t=j\}=\alpha(1-\delta)\theta_H^{t-j}(1-\theta_H)^j/D, \tag{7a}$$

$$\Pr\{\Psi=L | v_t=j\}=\alpha\delta\theta_L^{t-j}(1-\theta_L)^j/D, \tag{7b}$$

$$\Pr\{\Psi=0 | v_t=j\}=(1-\alpha)\theta_0^{t-j}(1-\theta_0)^j/D, \tag{7c}$$

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[6] If $h_I<h_U$ and $0<\gamma<1$, then the result follows trivially, while if $h_I=1$ and $0<\gamma<1$, then the result is reversed.

where $D=\alpha(1-\delta)\theta_H^{t-j}(1-\theta_H)^j+\alpha\delta\theta_L^{t-j}(1-\theta_L)^j+(1-\alpha)\theta_0^{t-j}(1-\theta_0)^j$. We first show that (7a) is an increasing function of j above a certain value of j by dividing the denominator by the numerator,yielding $1+\delta(\theta_L/\theta_H)^{t-j}[(1-\theta_L)/(1-\theta_H)]^j/(1-\delta) + (1-\alpha)(\theta_0/\theta_H)^{t-j}[(1-\theta_0)/(1-\theta_H)]^j / [\alpha\ (1-\delta)]$. It can be easily seen that this last quantity is a decreasing function of j beyond a certain value of j, since $\theta_H<\theta_L$ and $\theta_H<\theta_0$, implying that its inverse is an increasing function of j. Similarly, dividing (7b) by (7a), we get $\delta(\theta_L/\theta_H)^{t-j}[(1-\theta_L)/(1-\theta_H)]^j/(1-\delta)$ which, by the same reasoning, is also a decreasing function of j above a certain value.

Replacing now V^* and $\Pr\{\Psi=0\,|v_t=j\}=1-\Pr\{\Psi=H\,|v_t=j\}-\Pr\{\Psi=L\,|\,v_t=j\}$ in (6), we get the following:

$$E[V_{t+1}\,|\,v_t]=V^*+(V_H-V_L)\bullet\Pr\{\Psi=H\,|v_t=j\}\bullet[(1-\delta)-\delta\Pr\{\Psi=L\,|\,v_t=j\}/\Pr\{\Psi=H\,|\,v_t=j\}] \quad (8)$$

Examining this expression, we have the constant V^* plus $V_H - V_L$ (>0) times the product of (7a), which was shown to be increasing in j, and $(1-\delta) - \delta$ times the quotient of expressions (7b) and (7a); since the quotient was shown to be decreasing in j, for sufficiently small δ, the expectation will be increasing in j. The sufficient condition for δ follows :

$$[(1-\delta) - \delta\ \Pr\{\psi=L|\,v_t=j\}/ \Pr\{\psi=H\,|\,v_t=j\}] > 0$$

$$(1-\delta)^2 - \delta^2(\theta_L/\theta_H)^{t-j}[(1-\theta_L)/(1-\theta_H)]^j > 0$$

$$(1-\delta) > \delta\{(\theta_L/\theta_H)^{t-j}[(1-\theta_L)/(1-\theta_H)]^j\}^{1/2}$$

$$1 > \delta\,\{1 + [(\theta_L/\theta_H)^{t-j}[(1-\theta_L)/(1-\theta_H)]^j]^{1/2}\}$$

$$\delta < \{1 + [(\theta_L/\theta_H)^{t-j}[(1-\theta_L)/(1-\theta_H)]^j]^{1/2}\}^{-1}$$ **.QED.**

A sufficient condition on δ, the probability of a low signal, is that it be less than or equal to 1/2. The necessary condition of Theorem 1 is achieved for θ_L not much larger than θ_H or for $\mu(1-h_I)$ not too large; even when these are not true, δ still respects the condition when j approaches t. By definition $\theta_L > \theta_H$, so that $(1-\theta_L) < (1-\theta_H)$ and for large j the second ratio dominates the first ratio. The implication is that as δ increases and the intensity of informed traders wishing to be short-sellers increases, it takes more periods of trading to interpret the signal sent by increasing volume as positive. In this scenario, more no trades are likely to occur ceteris paribus; hence more actual trades, buys and sales, are needed to revise estimates of the asset value. Figure 2 a, b and c graphically exhibit the conditions and reasonable size of δ for Theorem 1 to hold. The conclusion is, for small δ, the value estimated by the market maker and the price set will rise as the trading volume increases, and fall as volume decreases. In the case of $h_I=h_U=1$ corresponding to EO, however, expected value is independent of volume; for $h_I=1$ we have $\theta_H=\theta_L$ and none of the conclusions holds. Again, it is necessary that informed traders would want to short sell but are restrained by the prohibition of short sales.

(Figures 2a, 2b and 2b about here)

Recall also the crucial condition on the propensity to short sell. We required that the propensity to short sell by uninformed investors be less than that of informed investors, or $(1- h_U)\gamma < (1 - h_I)$ which requires immediately that $h_I < 1$.

III. CHARACTERISTICS OF THE SPEED OF PRICE CONVERGENCE

Our intent is to add to the findings concerning the volume of trading on information, by examining the relative speed of adjustment of the stock price as it converges to the value V_H or V_L, whichever is perceived to be true. One may conjecture that the lower volume adjustment of the price on bad news occurs by larger and thus faster moves downwards than the slower, but more liquid, trading that moves the price up on good news. This is not necessarily the case, however; trading on bad news could occur as a discontinuous, infrequent pattern of trades. Whichever might be the case may well depend upon the composition of the market. By characterizing the trading pattern further, and basing the pattern on the proportions of types of traders and their stock holdings, we are able to generate some testable implications.

In order to measure speed of convergence, we shall use the statistical concept of *entropy*. Entropy[7] is a conceptual measure of the uncertainty of a random variable. One can interpret entropy as the expected value of the natural logarithm of the inverse of the probability of a random variable. In this case, the uncertainty is in the market maker's belief about the true state of nature described by the existence and type of signal that has occurred and the resulting value of the price. *Relative entropy* is defined as a measure of the 'distance' between two probability distributions. Mathematically, the relative entropy of $P^{\psi'}$ under P^{ψ} is defined as:

$$I_P^{\psi}\,(P^{\psi'}) = \sum_{Q \in \{N,B,S\}} P^{\psi}(Q) \ln [P^{\psi}(Q) / P^{\psi'}(Q)] \qquad (9)$$

This expression is used frequently in information theory due to its mathematical properties. By assuming the distribution of $P^{\psi'}$ when the true distribution is P^{ψ}, one can measure the relative distance or convergence between the two probability distributions over the sequential trades. The relative entropy is thus a measure of the inefficiency of that assumption and provides the rate of exponential convergence of probabilities and prices to their limits, which are presumed to be the true values.

The use of entropy to measure the exponential convergence is only valid for a Markov process; this property was earlier established for the counting processes for trades, leading to the distributions. The observed proportions of trades, which are the posterior distributions, would converge almost surely (a.s.) to the true equilibrium values by the Strong Law of Large Numbers. Since the equilibrium price is a linear function of the market maker's revised probabilities of the terminal values, the price would converge a.s. to V_{ψ} at the exponential rate determined by the

[7] Entropy was suggested by Easley and O'Hara (1992a) for this purpose; we extend their notion here by deriving the actual measures and comparing them under the different informational alternatives. Diamond and Verrecchia used the alternative of first passage time to measure convergence.

minimum of $I_P^{\psi}(P^0)$ and $I_P^{\psi}(P^H)$ for ψ = L, H. The following proposition captures this characterization and also defines which signal generates the minimum convergence rate for each case ψ.

Proposition 3: Transaction prices, converge to their strong form efficient values at exponential rates. If signal ψ occurs, the exponential rate of convergence is:

$$r(\psi) = \text{Min. } \{I_P^{\psi} (P^{\psi'}): \psi' \neq \psi\}, \text{ where } I_P^{\psi} (P^{\psi'}) \text{ is the entropy of } P^{\psi} \text{ relative to } P^{\psi'}.$$

$I_P^L (P^O)$ and $I_P^H (P^O)$ are the minima for ψ = L and H[8].

Proof: See the appendix.

Consequently, for the case of a high signal, ψ = H, the exponential rate of convergence is $I_P^H(P^O)$, as it is less than $I_P^H(P^L)$. For the low signal case, ψ = L, the rate of convergence is $I_P^L(P^O)$, as it is less than $I_P^L(P^H)$. Note that if h_U = 1 or 0, the results are indeterminate; if all or none of the uninformed traders own the stock, the market maker can deduce the signal occurrence in finite time (a.s.). For that reason, the speed of adjustment would explode since the market price would converge immediately to the true value. Using a more detailed analysis, both minima $I_P^H (P^O)$ and $I_P^L (P^O)$ from Proposition 3 are sensitive to the variation in γ and μ as shown in Figures 3 and 4. In fact, both convergence rates are increasing monotonically in μ and γ. As the probability of informed trader participation increases, the speed of convergence increases; the market maker learns the 'bad' news faster when confronted by numerous insiders subject to short sale constraints[9].

(Figures 3 and 4 about here)

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8 For Ψ = 0, the minimum rate of convergence is parameter dependent. For γ = ½ as our benchmark case, however, the minimum rate is $I^0_p (P^H)$ for all other parameter values. The proof is not included in the appendix, but is available on request from the authors.

9 A high proportion of insider ownership is likely to be true in entrepreneur-dominated business environments, as found in emerging markets such as Thailand.

The major result is that the market maker will adjust his price downward faster in the 'bad' news case, as he increases his belief that the trade originated with an insider. Consequently, when associated with lower trading volume in the low signal, prices can be expected to adjust at a higher speed than in the case of a high signal, leading to high volume. This is formalized in the following proposition, where the rate of convergence in the case of a low signal is compared to the rate for a high signal.

Proposition 4: In general, the rate of convergence of quoted prices is parameter dependent. When uninformed investors are equally likely to buy or sell (γ =1/2), however, and for all other parameter values, the convergence rates for ψ = L and ψ = H are equal if $h_I = h_U$ but faster for ψ = L, if $h_I \neq h_U$. That is,

$$\text{Min. }\{I_P^L (P^O), I_P^L (P^H)\} = \text{Min. }\{I_P^H (P^O), I_P^H (P^L)\} \quad \text{for } \gamma = 1/2;\ h_I = h_U \tag{10}$$

and

$$\text{Min. }\{I_P^L (P^O), I_P^L (P^H)\} > \text{Min. }\{I_P^H (P^O), I_P^H (P^L)\} \quad \text{for } \gamma = 1/2;\ h_I \neq h_U \tag{11}$$

Proof: See the appendix.

Some discussion of the parameter values is warranted. The standard assumption in previous studies is γ = 1/2, or that the uninformed is as likely to be a buyer as a seller. The proposition states that in this case and for $h_I \neq h_U$, the rate of convergence, $I_P^L (P^O)$, on 'bad' news is faster than $I_P^H (P^O)$, the rate on 'good' news. When $h_I = h_U$, the two convergence rates are equal; that is, the market maker learns less from the trade process in the unlikely event of equal proportions of informed and uninformed traders owning the stock. The convergence result in Proposition 3 still holds if the majority of uninformed want to *buy* (i.e., $\gamma < .5$) as depicted in Figure 5a; however, the inequality can be easily inverted in most cases when the majority of uninformed want to *sell* (i.e., $\gamma \geq 0.6$). The conclusion of Proposition 3 is robust relative to other parameter values (i.e., μ, h_I and h_U) when varying one parameter and holding the other parameters equal to one half, as shown in Figure 5b. As for Proposition 3, so also the result for Proposition 4 is indeterminate when h_U = 1 or 0. One may hypothesize that the degree of ownership is irrelevant in nearly all cases, unless the market maker knows with certainty whether the uninformed own the stock or not[10]. Hence, we claim that the speed of adjustment in the downward case is faster than in the upward case.

(Figures 5a and 5b about here)

The low signal case is, of course, the most important one, as it is the inability of the informed trader to profit from the privately observed signal that causes the asymmetry in trading. For that reason, we have investigated the reaction to a low signal as a function of the relative holdings of informed and uninformed investors. The results are summarized in:

Proposition 5: The rate of convergence for $\Psi = L$ is dependent on the relative levels of h_I and h_U. Specifically:

i) The rate of convergence is minimized for $h_I = h_U$.

ii) The rate of convergence is infinite for $h_U / h_I = 0$ or $(1- h_U)/(1- h_I) = 0$; that is, when $h_I \neq h_U$ and $h_U = 0$ or 1, the low signal is instantaneously recognized, as uninformed investors can always sell if present in the market.

iii) The rate of convergence decreases monotonically as h_U increases for $h_I = 1 - (1- h_U)\gamma$, the upper boundary by condition $(1- h_U)\gamma < (1 - h_I)$; hence, for low values of h_U, the fastest convergence occurs for high values of h_I.

iv) The rate of convergence increases monotonically as h_U increases for $h_I = 0$; hence, for high values of h_U, the fastest convergence occurs for low values of h_I.

Proof: See the appendix.

[10] In fact, on a high signal, $I_p^H(P^0)$ is independent of the value of the h_I parameter

We see that a rapid recognition of the signal by the market maker is impeded by having roughly equal ownership ratios by the informed and uninformed investors and facilitated by unequal ownership, particularly at extreme differences, which can cause immediate recognition.

The subject of the short sale restriction is a controversial one. One argument is that a short sale restriction is needed to prevent informed traders from abusing their material informational advantage over the uninformed traders. Another argument is that the short sale restriction may affect the information efficiency in the market, because constraining pessimists without constraining optimists imparts an upward bias to stock prices as suggested by Miller [1977] and Figlewski [1981]. As a result of the above findings and associating low ownership with an **effective** ban on short sales, we infer a positive effect from the prohibition of short sales.

Theorem 2: In general, the imposition of a short sale prohibition improves the informational efficiency of the market by expediting the convergence of a stock price to its equilibrium value when $\psi = L$ and leaving it unaffected when $\psi = H$[11].

Proof: Proposition 5 states that convergence improves with the asymmetry of ownership. In particular, the case $h_I = h_U = 1$ or full ownership by both types of investors minimizes the rate of convergence; hence this impedes the discovery of occurrence of information by the market maker, who learns due to the asymmetry combined with the short-sale prohibition.

The short sale prohibition does not affect the trading behaviour of informed traders when the signal is high, but does prevent some informed traders' selling activities in other cases. With the prohibition, the market participants learn the 'bad' news faster, particularly from 'no trade' events. Proposition 2 and Theorem 2 indicate that the short sale prohibition promotes market efficiency. Provided the parameters of the market composition are consistent with our conditions, we should expect to see the faster adjustment on bad news reflected in actual trading.

[11] For $\Psi = 0$, the speed of convergence is faster when a short sale constraint exists and $\gamma = ½$. The proof is not shown in the appendix, but is available on request.

IV. TESTS OF THE PREDICTIONS

The conclusion of Theorem 1 is that there exists what would normally be an anomaly under the efficient markets hypothesis, namely that increased volume of trading is indicative of superior returns, due to the predictable response of traders in the face of asymmetric information. In practice, the increasing return based on volume in the previous period would be recognized from the observation of an above average return on stocks exhibiting a relative increase in trading volume in the previous period. To test for this, comparisons have to be made with normal trading volume and between returns on assets. It is also a concern whether noise in trading will obscure the phenomenon, so that an adjustment for noise may be necessary. We considered the lag time for the volume increase to be reflected in increased returns, and also whether single periods or moving averages of volume and returns should be used.

To verify the conclusion, we used data from the Stock Exchange of Thailand, on which short-selling is prohibited, for the period January 1998 to December 1999. Sixty companies were available with relatively complete data for that period. For these companies, we obtained daily price and volume figures and calculated the average daily trading volume for each for this period. This was used to normalize the daily volume to provide a relative turnover statistic (T), defined as daily volume over average daily volume. These were then segregated into portfolio deciles by relative turnover, and the returns were calculated for each decile; this was equivalent to forming equally-weighted portfolios of stocks experiencing relatively high or low turnover with the returns from such portfolios being determined. An estimate was made of decile divisions for relative turnover across all stocks, by ranking within a single file the relative turnover of all sixty stocks on all days; then returns of stocks trading within the identified turnover intervals were aggregated, all of this on a day-to-day basis. Specifically, for each stock, the number of observations for which daily turnover fell in each of the deciles[12] was tabulated along with

[12] The number of observations for each stock clearly differed by stock within each decile, and even the average number of observations across stocks by decile was not constant; the intention was to choose deciles that included a reasonable number

statistics (min, max, mean and standard deviation) of the volume and returns for that decile. Those statistics were then aggregated across all stocks to determine the average return (and other parameters) for each turnover decile.

Statistical analysis

We considered that Theorem 1 would be confirmed by evidence of increasing next-day return with increasing turnover decile, and also by correlation between the individual stock turnovers and next-day returns. Strictly speaking, as the theorem refers to a short-term phenomenon for a trading period, we would have preferred to develop results from controlled intra-day data; given the data limitations, we were able to test only if the turnover effect is recognizable on a daily basis; this also would be valuable as a trading tool if proven to be true. Because of the noise present with daily trading volume and returns, correlation results of single day statistics were less conclusive. We chose instead to use a five-day moving average of volume to generate the relative turnover as indicative of an increasing trend in trading; we then associated this with a lagged five-day moving average of daily returns and determined the correlation of the two time series for each stock. Table 1 presents the summary of the 5-day moving average results for a one-day lag, showing the returns and correlations across various ranges.

Table 1: Moving Average Returns and Correlations for Portfolios with Various Turnover Levels (T)

Turnover range	All T	T > .625	T > .985	T > 1.27	T > 2.10
Max, min return	.0286, -.0012	.0239, -.0027	.0140, -.0014	.0328, -.0005	.0408, -.0041
Average return	.0038	.0101	.0140	.0165	.0219
Max, min correlation	.5388, .0316	.5928, -.0617	.5781, -.1272	.5944, -.2784	.6609, -.4361
Average correlation	.3032	.2675	.2352	.2053	.1704
Pos've, neg've corrlns	60,0	58,2	53,7	52,8	46,14
Avg. no. observations	461.5	230.4	153.4	115.4	57.7

of observations for each stock

In this case there is an obviously monotonic pattern. As the average turnover is restricted increasingly, the average return increases yet the correlation results deteriorate with the sample size, with both average correlation and the number of positive correlations decreasing. On the other hand, the prediction of Theorem 1 is verified, as it is clear that sixty out of sixty positive correlations is significant by any sign test. Table A2 (appendix) presents the correlations plus t and F statistics for the sixty stocks. The results range from the highest correlation of .5388 with an F-value of 193.5 and a t-statistic of 13.9 to the lowest correlation of .0316 with an F-value of .457 and a t-statistic of .676; the average correlation of .3032 has an F-value of 61.62 and a t-statistic of 7.06, which is clearly significant. Alternatively, all except the six companies with correlations less than .10 had highly significant (more than 99.5% level) correlation between relative volume and return, with all except twelve companies at more than 99.999% level significance.

The moving averages were associated on the basis of a single day lag (i.e., moving average of $\{vol_{t-4}, \ldots, vol_t\}$ associated with moving average of $\{rtn_{t-3}, \ldots, rtn_{t+1}\}$). This was also examined by trying two and three day lags for the five-day moving average on a sample of eight companies, some of which were chosen as high volume stocks; the correlations systematically decreased as the lag increased, the average correlation falling as .28, .21,.15 . We took this as strongly supportive of the theorem, as a greater lag would reflect less efficiency in the market reaction, and is not related to the noise treatment by averaging. We also investigated a shorter moving average of three days or none, finding that increasing to a five-day average monotonically improved the correlations as it corrected for noise problems; average correlations rose from .02 to .19 to .28 as the moving average increased for the sample. We noted that correlations increased with the volume of each security in this sample; that is, correlation score was correlated strongly (.8677) with the actual average volume traded for the eight companies.

For the non-moving average data, the relative turnover deciles, while based on the mean turnover of unity, were significantly skewed to the right, as may be expected. There is considerable evidence of both low and high trading, with a maximum turnover that averaged twenty times the mean in the highest decile. There were isolated examples of companies with extremely high one-day trading volumes, which were curtailed to an arbitrary 25 times average trading volume for the stock. We found that the median turnover was approximately .505, with the mean turnover occurring at approximately the eighth decile. (Note that because of averaging, the distribution of the one-day turnover is compressed, with its median lower than the .625 of the moving average statistics.)

Table 2: Single-day Returns and Correlations for Portfolios with Various Turnover Levels (T)

Decile	All vol.	I	II	III	IV	V	VI	VII	VIII	IX	X
Turnover	0<T<25	T<.071	.071<T<.15	.15<T<.242	.242<T<.36	.36<T<.505	.505<T<.703	.703<T<.993	.993<T<1.45	.145<T<2.46	T>2.46
Observations	465.5	47.6	46.4	46.7	47.2	46.1	46.7	46.8	46.4	47.0	46.3
Correlation	0.0481	0.0016	0.0593	0.0601	0.0639	0.0138	0.0588	-0.0192	0.0151	-0.0088	0.0080
Average T	1.0006	0.0403	0.1294	0.1952	0.2995	0.4282	0.5978	0.8386	1.1996	1.8805	4.4698
Std dev T	1.4602	0.0179	0.0223	0.0264	0.0341	0.0416	0.0570	0.0857	0.1302	0.2803	2.2149
Average Rtn	0.0036	-0.0026	0.0011	0.0031	0.0019	0.0025	0.0025	0.0053	0.0065	0.0091	0.0102
Std dev Rtn	0.0653	0.0558	0.0590	0.0576	0.0557	0.0570	0.0613	0.0610	0.0625	0.0760	0.0772

Table 2 summarizes the statistics associated with the decile analysis for the individual daily turnovers and returns. It reveals first that the average correlation of .0481 is significantly lower than that of .3032 for the moving average. More importantly, the average return rises steadily, if not monotonically, across the deciles. This result is seen best in Figure 6a, and further illustrated in Figure 6b where the deciles are cumulated (summing within the decile ranges changes summing up to the breakpoints); Figure 6b has a more obvious monotonic result. Note especially that the average return rises considerably in the upper deciles, with the top decile offering a daily average return of .91%. We checked also for other properties, finding that although the standard deviation of return rises for the upper deciles, the coefficient of variation quite steadily declines[13].

(Figures 6a, 6b about here)

We conclude from this analysis that for the period and data used, the turnover anomaly that was identified theoretically is strongly indicated by the high correlation exhibited by a moving average analysis, which appears to eliminate the noise present in single-day results. The returns associated with the rising deciles of single-day turnover also indicate a significant opportunity for trading profits from observing the trading patterns and investing in stocks which are actively traded relative to their norms. In the next section, we demonstrate a tradable strategy based on turnover observation and observe from its results that there may be more characteristics in the trade history than are revealed by the statistics above.

A trading simulation

In order to demonstrate the apparent anomaly as a successful approach to extraordinary gains, we used the data set to form portfolios based on the turnover of the individual stocks. This process selected stocks on a daily basis given only their relative trading volumes on a given day to select a portfolio held on the subsequent day. No adjustment has been made for systematic risk; relative gains are assessed only by reference to the null portfolio consisting of investment in all sixty stocks for the same period. Due to the short time period, no in and out of sample test was attempted. This meant that stocks were selected based on their ex post relative volume over the period, with those in their actual top decile of daily relative volume selected for the portfolio[14].

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[13] A further analysis regressed same day returns against relative volume, finding much higher correlation; this result is certainly compatible with an intra-day response, but is not a test of the trading interval hypothesis

[14] In actual trading, one would have to have determined previously the volume distribution and to have identified what

The comparison portfolio was formulated as an equally-weighted fund composed of an investment of $1000 in each of the sixty stocks. This portfolio was assessed for its daily returns in two ways, one being a true buy-and-hold, while the other was actually re-weighted equally each day. The active portfolio was formed by equal investments in whichever stocks traded in their top deciles respectively on each day, leading to 74 days on which no investment was possible and a T-bill return was generated. Over the two-year period, an average of 6.85 stocks were held in the portfolio on those days where it was not empty, with a maximum of 45 stocks and a standard deviation of 7.42; the portfolios (not the individual stocks) had a maximum daily return of 28.94%, a minimum of –22.22%, and a mean daily return of .0944% with a standard deviation of 5.72%.

We should caution at this point that the SET was in the midst of a major recovery. The total gain from the active portfolio formed from the top decile was 622%, with a final value of $438,240 from the initial $60,000 invested; this must be kept in perspective. During the two-year period, the $60,000 invested in the sixty stocks of the null portfolio rose to $238,219 on a buy-and-hold basis and to $186,880 if rebalanced daily to an equally-weighted holding; these represent gains of 297% and 211%, respectively. Perhaps more phenomenally, the active portfolio had reached a value of $637,060 (over 960% gain) on June 30, 1999 before falling precipitously to $302,090 in the next six weeks[15]. The null portfolio had similar gyrations as is revealed by Figure 7a, which demonstrates the obvious correlation between the day-to-day results of the active portfolio and the other portfolios; Figure 7b shows the net gains for the active portfolio over the two-year period.

(Figures 7a, 7b about here)

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would be considered as a top decile level of trading on any day

15 The determination of the returns was very sensitive to the method of calculation for the average returns; the geometric averages used gave much lower returns than the arithmetic averages.

It appears from Figure 7a that the active portfolio exhibited more volatility than the null portfolios, and one might speculate that the high returns are associated with high risk, particularly considering the lack of diversification in the portfolio, as suggested above; on several occasions, the portfolio consisted of a single security. The beta for the active portfolio was 1.06 over the period, so there does not appear to be any requirement for adjustment for systematic risk, but the lack of diversification does raise the issue of the correct risk measurement. The Sharpe measure is appropriate for comparison, if the only holding is the active decile portfolio; The Sharpe measure for this portfolio, estimated on daily returns, was .1005, in comparison to that of the null portfolio of .0763. It is also to be noted that the returns are prior to trading costs, and that these costs are high due to almost one hundred percent turnover in the portfolio on a daily basis.

Another consideration is the previously mentioned note that the SET was behaving extremely strongly during this period (especially during the nine-month period from October ’98 to July ’99). It is clear that the extraordinary gains of the top decile portfolio are due to the overall market performance; it could be questioned whether the performance might be reversed in a down market. Correlations of volume with return, however, do not support this conjecture, and the statistical results earlier support the link to the theoretically demonstrated cause and effect.

V. CONCLUSION

This paper has presented a theoretical proposal that a market anomaly of excess returns associated with high relative volume can be justified by the response of market participants to the suspicion of inside information used to affect buying and selling of individual stocks. Because of a prohibition on short-selling, that prevents insiders from selling stock following the occurrence of bad news, investors can deduce the bad news by recognition of no-trade events as indications of desired sales. The correlation of high volume with good news followed from this reasoning, as demonstrated in Theorem 1. It was further shown that the price adjustment to a new equilibrium is likely to be faster on bad news than on good news, provided certain conditions on market parameters were true, as summarized in Theorem 2.

The first of these theoretical results have been demonstrated empirically by considering recent data from the Stock Exchange of Thailand, which was used due to its absolute prohibition of short-selling. The data illustrated both the high correlation between one day's relative volume and the following day's returns and the increasing returns that are realized if the daily turnover is divided into deciles and those deciles' returns are analysed. As a demonstration of the success of a turnover-based strategy, the data used for the statistical analysis were used also in a simulation of the potential profits from trading on turnover information. It is somewhat remarkable that the results are generated from a daily strategy, even though the theoretical approach refers only to successive trading periods. This indicates a persistence in the phenomenon that should certainly be labelled as anomalous. The data can perhaps be questioned on the basis of the particular time period, during which the universe considered had a total return of almost 300%, but the statistical analysis did not indicate any dependence between the superiority of the turnover portfolio returns and the general direction of the market.

Investigation of the speed of adjustment to new information requires a detailed analysis of the market parameters, as was done in the papers of EKO (1997a) and EKOP (1996), since the results are conditional on the relationships between the parameters. The authors have conducted a preliminary study on this aspect, as reported in HRKP (1999) using a different data set; this study showed some consistency in parameters estimated, but also found that the limited trading in the SET made analysis difficult. Its finding was logically consistent with the prediction, but not a proof of it, in that both the conditions of Theorem 2 and the conclusion proved to be false. Potentially, a different data set that satisfied the conditions might also confirm the result.

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TABLE A1: A summary of notation used throughout the paper

Notation	Definition
V	The value of the risky asset per share. $V_H \equiv E\{V \mid Y = H\}$; $V_L \equiv E\{V \mid Y = L\}$; $V^* \equiv E\{V \mid Y = 0\}$ where $V_H > V^* > V_L$.
Y	a signal that V is either high (H), low (L) or no signal (0). $Y \in [0, H, L]$
α	The probability that an information event occurs. $0<\alpha<1$.
δ	The probability that a low signal $Y = L$ occurs. $0<\delta<1$.
γ	The probability that an uninformed trader wants to sell. $0<\gamma<1$.
μ	The probability that a given trader is informed; also the fraction of informed traders in the market. $0<\mu<1$.
h_I	The probability that a given informed trader already owns the stock. $0<h_I<1$.
h_U	The probability that a given uninformed trader already owns the stock. $0<h_U<1$.
B	The event that a trader buys a single unit quantity.
S	The event that a trader sells a single unit quantity.
N	The event that no trade occurs.
τ	A trade event occurs as either buy or sell. $\tau = \{B, S\}$.
n_t	The total of no trade events up to time t.
β_t	The total buying volume of the traded asset up to time t.
s_t	The total selling volume of the traded asset up to time t.
v_t	The total trading volume of the traded asset up to time t. $v_t = b_t + s_t$.
Q_t	The event of a buy, a sell or no trade at time t. $Q_t \in [B_t, S_t, N_t]$.
Q^{t-1}	The sequence (vector) of past trading outcomes. $Q^{t-1} = [Q_1, Q_2, \ldots, Q_{t-1}]$.
$\rho_{\psi,t}$	The conditional probability of signal $\psi \in [H,L,0]$, given trade history $Q^{t-1} = [Q_1, Q_2, \ldots, Q_{t-1}]$. (i.e. $\rho_{\psi,t} \equiv \Pr.\{\psi \mid Q^{t-1}\}$)
$\rho^{Q}_{\psi,t+1}$	The conditional probability of signal $\psi \in [H,L,0]$, given one more trade, $Q_t \in [B_t, S_t, N_t]$ following trade history, $Q^{t-1} \equiv [Q_1, Q_2, \ldots, Q_{t-1}]$.
θ_ψ	The conditional probability of a no trade event given signal ψ.
$I_{P^\Psi}(P^{\Psi'})$	The relative entropy under probability law $P^{\Psi'}$ given that $P^{\Psi} = \Pr.\{Q \mid \Psi\}$ is true.

Table A2 - Summary of correlations between relative volume and returns for 5-day moving average and single-day values, and average daily return (Data from 01-Jan-98 - 31-Dec-99).

	5-day moving average				Single-day		
STOCK	Correlation	t-stat	F	Significance	Correlation	Avg. return	Observations
ADVANC9	0.0718	1.5666	2.4542	0.117880	-0.0602	0.195%	479
AFC9	0.1947	3.6112	13.0409	0.000352	0.1058	0.462%	337
ASL9	0.1865	4.1275	17.0359	0.000043	-0.0506	0.585%	479
AST9	0.2998	6.8140	46.4311	0.000000	0.0615	0.484%	476
ATC9	0.4446	10.7938	116.5052	0.000000	0.0130	0.456%	479
BANPU9	0.2889	6.4712	41.8761	0.000000	0.0781	0.031%	466
BAY9	0.4676	11.5050	132.3654	0.000000	0.1005	0.141%	479
BBL9	0.4075	9.7052	94.1916	0.000000	0.1238	0.070%	479
BCP9	0.4125	9.8487	96.9971	0.000000	0.0676	0.248%	479
BEC9	0.2033	4.5120	20.3582	0.000008	0.0760	0.128%	478
BECL9	0.2442	5.4778	30.0059	0.000000	0.0391	0.008%	479
BIGC9	0.2999	6.6394	44.0817	0.000000	0.0095	0.381%	452
BLAND9	0.2613	5.8803	34.5779	0.000000	-0.0006	0.533%	478
BOA9	0.3483	8.0738	65.1855	0.000000	0.0532	0.348%	478
CAPE9	0.3062	6.9962	48.9471	0.000000	-0.0109	-0.030%	479
CNS9	0.4749	11.7364	137.7441	0.000000	0.0596	0.555%	479
COCO9	0.2002	4.4444	19.7524	0.000011	0.0755	-0.055%	479
DELTA9	0.1432	3.1364	9.8370	0.001818	0.0438	0.154%	476
DTDB9	0.5083	12.8379	164.8114	0.000000	0.1490	0.234%	479
EGCOMP9	0.0457	0.9952	0.9905	0.320133	-0.0670	-0.091%	479
HANA9	0.1721	3.7959	14.4090	0.000166	0.0477	0.206%	478
HEMRAJ9	0.1522	2.8402	8.0668	0.004780	-0.0029	-0.041%	346
IFCT9	0.4744	11.7211	137.3832	0.000000	0.0801	0.350%	479
ITD9	0.2921	6.5993	43.5503	0.000000	0.0522	0.452%	473
JASMIN9	0.3324	7.6643	58.7413	0.000000	0.1037	0.339%	479
JUTHA9	0.3721	7.1266	50.7888	0.000000	0.0741	0.339%	322
KK9	0.3990	8.8643	78.5766	0.000000	0.0848	0.710%	421
KTB9	0.4653	11.4318	130.6868	0.000000	0.0715	0.298%	479
NFS9	0.3695	8.6494	74.8124	0.000000	0.0875	0.380%	479
NPC9	0.5388	13.9098	193.4817	0.000000	0.1461	0.216%	479

Table A2 (cont'd)

	5-day moving average				Single-day		
STOCK	Correlation	t-stat	F	Significance	Correlation	Avg. return	Observations
NSM9	0.1780	3.9092	15.2817	0.000106	0.0099	0.354%	473
PTTEP9	0.0731	1.5940	2.5410	0.111593	0.0455	-0.049%	479
QH9	0.4275	10.2749	105.5726	0.000000	0.0560	0.585%	478
SAFARI9	0.0316	0.6761	0.4571	0.499301	-0.0760	0.020%	463
SATTEL9	0.5060	12.7445	162.4214	0.000000	0.1085	0.583%	478
SCB9	0.4643	11.4022	130.0102	0.000000	0.1551	0.118%	479
SCC9	0.4780	11.8241	139.8092	0.000000	0.1755	0.364%	478
SCCC9	0.3765	8.7836	77.1511	0.000000	0.0836	0.450%	473
SGACL9	0.2573	5.7798	33.4061	0.000000	-0.0687	0.433%	477
SGF9	0.2950	6.5937	43.4775	0.000000	0.0051	0.681%	462
SHIN9	0.1672	3.6807	13.5473	0.000260	0.0203	0.616%	477
SICCO9	0.0683	1.4040	1.9712	0.161064	0.0179	0.806%	426
SONE9	0.3011	6.8666	47.1504	0.000000	0.0298	0.524%	479
SPL9	0.4250	10.2114	104.2736	0.000000	0.0460	1.027%	479
SSI9	0.2032	4.5133	20.3701	0.000008	-0.0163	0.715%	479
SUC9	0.1984	4.3837	19.2167	0.000014	-0.0416	-0.094%	475
SUPALI9	0.3709	8.6573	74.9496	0.000000	0.0479	0.747%	476
TA9	0.3741	8.7734	76.9718	0.000000	0.0331	0.440%	479
TASCO9	0.2263	5.0478	25.4805	0.000001	0.0820	0.105%	478
TFB9	0.2658	5.9955	35.9462	0.000000	-0.0083	0.235%	479
THAI9	0.1899	4.2061	17.6912	0.000031	0.0126	0.102%	479
TISCO9	0.4360	10.5366	111.0199	0.000000	0.0482	0.351%	479
TMB9	0.4752	11.7466	137.9815	0.000000	0.0966	0.257%	479
TPI9	0.2750	5.9991	35.9887	0.000000	0.0316	0.600%	446
TPIPL9	0.3744	8.5281	72.7279	0.000000	0.0386	0.723%	452
TTA9	0.0868	1.7655	3.1171	0.078217	0.0040	0.255%	417
TYONG9	0.3114	7.1270	50.7940	0.000000	0.0163	0.503%	479
UBC9	0.4210	10.0741	101.4868	0.000000	0.1606	0.501%	477
UCOM9	0.4193	10.0129	100.2579	0.000000	0.1159	0.267%	476
ZMICO9	0.1360	2.9086	8.4599	0.003811	0.0447	1.173%	455
Max	0.5388	13.9098	193.4817	0.499301	0.1755	1.173%	479
Min	0.0316	0.6761	0.4571	0.000000	-0.0760	-0.094%	322
Average	0.3032	7.0633	61.6203	0.021660	0.0481	0.358%	465.5

Appendix

Proof of Proposition 1:

In the case of a no trade event, we need the expressions for $\rho^{N}_{*,t+1}$.

a) No News: $\rho^{N}_{0,t+1}$

$\Pr\{\psi = 0 | Q^{t-1}, N\} = \Pr\{\psi = 0 | Q^{t-1}\}\Pr\{N | \psi = 0\}/ [(A)+(B)+(C)]$

where $(A) = \Pr\{\psi = 0 | Q^{t-1}\}\Pr\{N | \psi = 0\}$

and $(B) = \Pr\{\psi = H | Q^{t-1}\}\Pr\{N | \psi = H\}$

and $(C) = \Pr\{\psi = L | Q^{t-1}\}\Pr\{N | \psi = L\}$

$\Pr\{\psi = 0 | Q^{t-1}, N\} =$

$$\rho_{0,t}\,\gamma(1- h_U)/\{\rho_{0,t}\gamma(1- h_U)+\rho_{H,t}(1-\mu)\gamma(1- h_U)+\rho_{L,t}[(1- h_U)\gamma(1-\mu)+\mu(1- h_I)]\}$$

$$= \rho_{0,t}\gamma(1- h_U) / [\rho_{0,t}\,\gamma(1- h_U)+(1-\mu)\gamma(1- h_U)(1-\rho_{0,t}) +\mu(1-h_I)\rho_{L,t}]$$

If $h_I = 1$, the multiplier of $\rho_{0,t}$ will be greater than 1, as $(1-\rho_{0,t}) > 0$; if $h_I = h_U$, however, the same result requires that $-(1-\rho_{0,t}) + \rho_{L,t}(1/\gamma)<0$, which is possible, but not necessarily true. Hence we claim the result is parameter dependent.

b) Low Signal: $\rho^{N}_{L,t+1}$

$\Pr\{\psi = L | Q^{t-1}, N\} = \Pr\{\psi = L | Q^{t-1}\}\Pr\{N | \psi = L\}/ \Pr\{N\}$

$$= \rho_{L,t}\,[\mu(1-h_I) +(1-\mu)\gamma(1- h_U)] / [\rho_{0,t}\,\gamma(1- h_U)+(1-\mu)\gamma(1- h_U)(1-\rho_{0,t}) +\mu(1-h_I)\rho_{L,t}]$$

If $h_I = 1$, then the r.h.s. $= [\rho_{L,t}(1-\mu) / (1-\rho_{0,t})(1-\mu) + \rho_{0,t}] < \rho_{L,t}$; but if $h_I = h_U$, then the multiplier of $\rho_{L,t}$ reduces to $[\mu + (1-\mu)\gamma] / [\rho_{0,t}\,\gamma + (1-\mu)\gamma(1-\rho_{0,t}) +\mu\,\rho_{L,t}]$, which is less than 1 iff $\mu(1-\rho_{L,t} - \gamma\rho_{0,t}) < 0$. Since this is not necessarily true, $\rho_{L,t+1}$ may exceed $\rho_{L,t}$ and we claim the result is parameter dependent.

c) High Signal: $\rho^{N}_{H,t+1}$

$$\Pr\{\psi = H | Q^{t-1}, N\}= \rho_{H,t}(1-\mu)\gamma(1- h_U) / \rho_{0,t}\,\gamma(1- h_U) +\rho_{H,t}(1-\mu)\gamma(1- h_U)+ \rho_{L,t}[(1- h_U)\gamma(1-\mu)+\mu(1- h_I)]$$

Since $0 < (1- h_I)\rho_{L,t} + \gamma(1- h_U)\rho_{0,t}$, we have

$(1-\mu)\gamma(1-h_U) < \gamma(1- h_U)[1-\mu(1-\rho_{0,t})]+ \mu(1- h_I)\rho_{L,t}$ and $\rho_{H,t+1} < \rho_{H,t}$. **QED**

Proof of Corollary: In the no trade case:

$$\frac{r^{N}_{L,t+1}}{r^{N}_{H,t+1}} = \frac{r^{N}_{L,t}\left[m(1-h_I)+(1-m)g(1-h_U)\right]}{r^{N}_{H,t}\left[(1-m)g(1-h_U)\right]}$$

Since $[\mu(1-h_I)+(1-\mu)\gamma(1-h_U)] > [(1-\mu)\gamma(1-h_U)]$, $[\rho^N_{L,t+1} / \rho^N_{H,t+1}] > [\rho^N_{L,t} / \rho^N_{H,t}]$.

Similarly, in the sell case:

$$\frac{\rho^S_{L,t+1}}{\rho^S_{H,t+1}} = \frac{\rho^S_{L,t}\left[\mu h_I + (1-\mu)\gamma h_U\right]}{\rho^S_{H,t}\left[(1-\mu)\gamma h_U\right]}$$

Since $[\mu h_I + (1-\mu)\gamma h_U] > [(1-\mu)\gamma h_U]$, $[\rho^S_{L,t+1} / \rho^S_{H,t+1}] > [\rho^S_{L,t} / \rho^S_{H,t}]$.

$$\frac{\rho^B_{L,t+1}}{\rho^B_{H,t+1}} = \frac{\rho^B_{Lt}\,(1-\mu)(1-\gamma)}{\rho^B_{Ht}\,(1-\mu)(1-\gamma)+\mu}$$

In the buy case:

Since $[(1-\mu)(1-\gamma)] < [(1-\mu)(1-\gamma)+\mu]$, $[\rho^B_{L,t+1} / \rho^B_{H,t+1}] < [\rho^B_{L,t} / \rho^B_{H,t}]$. **QED.**

Proof of Proposition 3: We denote by $\rho_{\psi,t+1}$ the probability estimated by the market maker of the occurrence of signal ψ given observation of the trading history. We have

$$\rho_{\psi,t+1} = \Pr\{\psi \mid Q^t\} = \Pr.\{\psi \mid (s_t, b_t, n_t)\}, \quad \text{where } t = s_t + b_t + n_t,$$

since the process is Markov and only the number of trades, rather than the order of them, is of consequence. As in the proof of proposition 1, and by Bayesian revision,

$$\rho_{0,t+1} = \frac{\Pr\{\psi = 0\}\Pr\{Q^t = (s_t, \beta_t, n_t) \mid \psi = 0\}}{\Pr\{H\}\Pr\{Q^t \mid H\} + \Pr\{L\}\Pr\{Q^t \mid L\} + \Pr\{0\}\Pr\{Q^t \mid 0\}}$$

Let us then define for convenience, the terms:

$NUM_0 = (1-\alpha)(\gamma h_U)^{st}(1-\gamma)^{bt}[\gamma(1-h_U)]^{nt}$

$NUM_L = \alpha\delta[\mu h_I + (1-\mu)\gamma h_U]^{st}[(1-\mu)(1-\gamma)]^{bt}[\mu(1-h_I) + (1-\mu)\gamma(1-h_U)]^{nt}$

$NUM_H = \alpha(1-\delta)[(1-\mu)\gamma h_U]^{st}[\mu + (1-\mu)(1-\gamma)]^{bt}[(1-\mu)\gamma(1-h_U)]^{nt}$

$DEN = NUM_0 + NUM_L + NUM_H$

Then we can write,

$$\rho_{0,t+1} = NUM_0 / DEN$$

$$\rho_{L,t+1} = NUM_L / DEN$$

and $$\rho_{H,t+1} = NUM_H / DEN$$

$$\therefore \ln [\rho_{0,t+1} / \rho_{L,t+1}] = \ln [(NUM_0 / DEN) / (NUM_L / DEN)] = \ln [NUM_0 / NUM_L]$$

$$= \ln [(1-\alpha)/\alpha\delta] + s_t \ln (\gamma h_U) + b_t \ln(1-\gamma) + n_t \ln [\gamma(1- h_U)]$$

$$-\{s_t \ln[\mu h_I+(1- \mu)\gamma h_U]+b_t \ln[(1- \mu)(1-\gamma)]+ n_t \ln[\mu(1- h_I)+(1- \mu)\gamma(1- h_U)]\}$$

Since $\lim_{t \to \infty} 1/ t \ln [\rho_{0,t+1} / \rho_{L,t+1}] = \lim_{t \to \infty} 1/(s_t+b_t +n_t) \ln [\rho_{0,t+1} / \rho_{L,t+1}]$, we have

$$\lim_{t \to \infty} \ln[\rho_{0,t+1}/\rho_{L,t+1}] / t= \lim_{t \to \infty}\{\ln[(1-\alpha)/\alpha\delta]/t + s_t /t \ln[\gamma h_U] + b_t/t \ln[1-\gamma]+ n_t/t \ln [\gamma(1- h_U)]$$

$$-\{s_t/t \ln[\mu h_I+(1- \mu)\gamma h_U] + b_t/t \ln[(1- \mu)(1-\gamma)] + n_t/t \ln[\mu(1- h_I)+(1- \mu)\gamma(1- h_U)]\} \quad (1)$$

Suppose signal L occurred. Then by the SLLN, s_t/t , b_t/t and n_t/t would converge a.s. to $P^L(S)$, $P^L(B)$ and $P^L(N)$, respectively, in the long run (i.e. to $[\mu h_I + (1- \mu)\gamma h_U]$, $[(1- \mu)(1-\gamma)]$, $[\mu(1- h_I)+(1- \mu)\gamma(1- h_U)]$). Note also that $1/t \ln[(1-\alpha)/\alpha\delta] \to 0$, Then, the above limit is the rate of exponential convergence:

$$\mathbf{S}_{Q\in\{B,S,N\}} P^L(Q) \ln [P^0(Q)] - \mathbf{S}_{Q\in\{B,S,N\}} P^L(Q) \ln [P^L(Q)] = - I_P{}^L (P^0) < 0.$$

Similarly, if L occurred

$$\lim_{t \to \infty} 1/t \ln [\rho_{H,t+1} / \rho_{L,t+1}] = \lim_{t \to \infty} [\{\ln[\alpha(1-\delta)/\alpha\delta]/ t + s_t /t \ln[(1- \mu)\gamma h_U] +$$

$$b_t/t \ln[\mu+ (1- \mu)(1-\gamma)] + n_t/t \ln [(1-\mu)\gamma(1- h_U)] \}-\{s_t/t \ln[\mu h_I+(1- \mu)\gamma h_U] +$$

$$b_t/t\ln[(1- \mu)(1-\gamma)] + n_t/t \ln[\mu(1- h_I)+(1- \mu)\gamma(1- h_U)]\}] \quad (2)$$

Hence, $\mathbf{S}_{Q\in\{B,S,N\}} P^L(Q) \ln [P^H(Q)] - \mathbf{S}_{Q\in\{B,S,N\}} P^L(Q) \ln [P^L(Q)] = - I_P{}^L (P^H) < 0.$

Finding the minimum of $\{I_P{}^L (P^H), I_P{}^L (P^0)\}$ is equivalent to determining the minimum of the first terms in (1) and (2), since the second terms are identical. Thus, we have:

$$I_P{}^L (P^0) = P^L(S) \ln [P^L(S)/ P^0(S)] + P^L(B) \ln [P^L(B)/ P^0(B)] + P^L(N) \ln [P^L(N)/ P^0(N)]$$

$$= P^L(S)\, ln\left[(1-m) + \frac{mh_I}{gh_U}\right] + P^L(B)\ln(1-m) + P^L(N)\ln\left[(1-m)+\frac{m(1-h_I)}{g(1-h_U)}\right] \quad (3)$$

$$= I_P{}^L\left(P^H\right) = P^L(S)\, ln\left[1+\frac{mh_I}{(1-m)gh_U}\right] + P^L(B)\ln\left[\frac{(1-m)(1-g)}{m+(1-m)(1-g)}\right]$$

$$+ P^L(N)\ln\left[1+\frac{m(1-h_I)}{(1-m)g(1-h_U)}\right] \quad (4)$$

$$= [\mu + (1-\mu)\gamma]\ln(1-\mu) + (1-\mu)(1-\gamma)\ln\left[\frac{\mu}{1-\gamma} + (1-\mu)\right]$$

$$= [(\mu + \gamma - \mu\gamma) + (1-\mu-\gamma+\mu\gamma)]\ln(1-\mu) + (1-\mu)(1-\gamma)\ln\left[1 + \frac{\mu}{(1-\mu)(1-\gamma)}\right]$$

$$= \ln(1-\mu) + (1-\mu)(1-\gamma)\ln\left[1 + \frac{\mu}{(1-\mu)(1-\gamma)}\right] \qquad (5)$$

$$\Delta = I^L{}_P\left(P^0\right) - I_P{}^L\left(P^H\right) = \left(\mu\eta_I + (1-\mu)\gamma\eta_U\right)1n(1-\mu) + (1-\mu)(1-\gamma)1n\left[\frac{\mu}{1-\gamma} + 1 - \mu\right]$$

$$+ \left[\mu(1-\eta_I) + (1-\mu)\gamma(1-\eta_U)\right]\ln(1-\mu)$$

$$= \left(\mu\eta_I + \mu(1-\eta_I)\right) + \left((1-\mu)\gamma\left(\eta_U + (1-\eta_U)\right)\right)\ln(1-\mu) + (1-\mu)(1-\gamma)\ln\left[\frac{\mu}{1-\gamma} + 1 - \mu\right]$$

At $\mu = 0$, $\Delta = 0$ since ln (1) + 1 (1-γ) ln (1+0) = 0, while at $\mu = 1$, $\Delta = -\infty$ since

$$\lim_{\mu\to 1} \ln(1-\mu) + (1-\mu)(1-\gamma) \lim_{\mu\to 1} \ln\left[1 + (\mu/(1-\mu)(1-\gamma))\right] = -\infty$$

Taking the partial derivative, we have:

$$\frac{\partial\Delta}{\partial\mu} = \frac{-1}{1-\mu} - (1-\gamma)\ln\left[1 + \frac{\mu}{(1-\mu)(1-\gamma)}\right] + (1-\mu)(1-\gamma)\frac{\partial}{\partial\mu}\ln\left[1 + \frac{\mu}{(1-\mu)(1-\gamma)}\right]$$

$$= \frac{-1}{1-\mu}\left[1 + (1-\mu)(1-\gamma)\right]\ln\left[1 + \frac{\mu}{(1-\mu)(1-\gamma)}\right]$$

$$+ (1-\mu)(1-\gamma)\frac{(1-\mu)(1-\gamma)}{\mu + (1-\mu)(1-\gamma)}\frac{(1-\mu)(1-\gamma) + \mu(1-\gamma)}{(1-\mu)^2(1-\gamma)^2}$$

$$\frac{\partial\Delta}{\partial\mu} = \frac{-1}{1-\mu}\left[1 + (1-\mu)(1-\gamma)\right]\ln\left[1 + \frac{\mu}{(1-\mu)(1-\gamma)} + \frac{1-\gamma}{\mu + (1-\mu)(1-\gamma)}\right] \qquad (6)$$

It is evident that at $\mu = 0$, (6) = 0 and at $\mu = 1$, (6) = $-\infty$. In fact, (6) > 0 iff

$$\frac{-1}{1-\mu}\left\{1 + (1-\mu)(1-\lambda)\ln\left[1 + \frac{\mu}{(1-\mu)(1-\gamma)}\right]\right\} < \frac{1-\gamma}{\mu + (1-\mu)(1-\gamma)}$$

At $\mu = 0$, both sides are equal, while the LHS of the relation is increasing in μ and the RHS decreasing in μ, implying that (6) <0 for all $\mu \in (0,1)$; hence, (6) ≤ 0 for all μ. Finally, therefore, given a low signal, $I_P{}^L(P^0) < I_P{}^L(P^H)$; that is, $I_P{}^L(P^0)$ is the slower rate of convergence.

Similar considerations apply to the case of a high signal, where we compare $I_P^H(P^0)$ and $I_P^H(P^L)$. We have:

$$I_P^H\left(P^0\right) = P^H(S)\ln(1-m) + P^H(B)\ln\left[\frac{m}{1-g}+(1-m)\right] + P^H(N)\ln[1-m] \quad (7)$$

$$I_P^H\left(P^L\right) = P^H(S)\ln\left[\frac{mh_I+1}{gh_U(1-m)}\right] - P^H(B)\ln\left[\frac{(1-m)(1-g)}{(1-m)(1-g)+m}\right] - P^H(N)\ln\left[\frac{m(1-h_I)+1}{(1-m)g(1-h_U)}\right] \quad (8)$$

$$\Delta = I_P^H\left(P^0\right) - I_P^H\left(P^L\right) = (1-m)gh_U \ln\left[\frac{mh_I}{gh_U}+(1-m)\right] + [(1-m)(1-g)+m]\ln(1-m)$$
$$+(1-m)g(1-h_U)\ln\left[\frac{m(1-h_I)}{g(1-h_U)}+(1-m)\right] \quad (9)$$

As before, at $\mu = 0$, $\Delta = 0$, and at $\mu=1$, $\Delta = -\infty$. Let us define $A \equiv (1-\mu)\gamma(1-h_U)$ and $B \equiv (1-\mu)\gamma h_U$, so that

$$\Delta = A \ln[1 + \mu(1-h_U)/A] + B \ln[1+ \mu h_I/B] + \ln(1-\mu)$$

Considering Δ as a function of μ only, we find its derivative, $d\Delta / d\mu$, using the chain rule

$$\frac{d\Delta}{dm} = \frac{\partial\Delta}{\partial m} + \frac{\partial\Delta}{\partial A}\frac{\partial A}{\partial m} + \frac{\partial\Delta}{\partial B}\frac{\partial B}{\partial m}$$

We observe that $\partial A/\partial\mu$ and $\partial B/\partial\mu$ are < 0. Also,

$$\frac{\partial\Delta}{\partial A} = \ln\left[1+\frac{m(1-h_U)}{A}\right] - \frac{A^2}{[A+m(1-h_I)]}\frac{[m(1-h_U)]}{A^2}$$
$$= \ln[1+w] - w/(1+w), \; where\, w = m(1-h_U)/A$$

and we recognize that this function is always positive.

Similarly, $\partial\Delta / \partial B > 0$. To show that $d\Delta/d\mu < 0$, it suffices, therefore, to show that $\partial\Delta/\partial\mu < 0$ for all (μ, A, B) such that $A, B < \gamma(1-h_U)$, $A + B < 1-\mu$.

$$\frac{d\Delta}{dm} = \frac{-1}{1-m} + \frac{A(1-h_I)}{A+m(1-h_I)} + \frac{Bh_I}{B+mh_I}$$

This expression is again a function of h_I, equal to $A / (A+\mu)$ for $h_I = 0$ and $B / (B+\mu)$ for $h_I = 1$, less $1/(1-\mu)$, and is concave, reaching its maximum at the value of h_I where

$$\frac{B}{A} = \frac{B+mh_I}{A+m(1-h_U)} \; or \; \frac{B}{B+mh_I} = \frac{A}{A+m(1-h_I)}$$

and $B(1-h_I) = A h_I$, implying $h_I = B / (A+B) = h_U$. Substituting, we find $\partial\Delta / \partial\mu < 0$ at $h_I = h_U$, implying $\Delta < 0$ always.

To conclude, therefore, given a high signal, $I_P^H(P^0) < I_P^H(P^L)$; that is, $I_P^H(P^0)$ is the slower rate of convergence. **QED.**

Proof of Proposition 4: We have shown that the convergence of the probability of news to sure values (0 or 1) occurs at the rates $r(\psi=L) = I_P^L(P^0)$ and $r(\psi=H) = I_P^H(P^0)$, implying that the distance between the presumed and the actual distributions is a.s. less than $e^{-r(\psi)}$. The two rates are given by:

$$I_P^L(P^0) = P^L(S)\ln\left[(1-\mu)+\frac{\mu h_I}{\gamma h_U}\right]+P^L(B)\ln(1-\mu)+P^L(N)\ln\left[(1-\mu)+\frac{\mu(1-h)_I}{\gamma(1-h)_U}\right] \quad (10)$$

$$I_P^H(P^0) = P^H(S)\ln(1-\mu)+P^H(B)\ln\left[\frac{\mu}{1-\gamma}+(1-\mu)\right]+P^H(N)\ln(1-\mu) \quad (11)$$

At $\gamma = 0$, (10) > (11) and at $\gamma = 1$, (11) > (10).

For $\gamma = ½$ as here assumed, let us define $L(h_I, h_U) \equiv$ (10) - (11); then

$$\begin{aligned} L(h_I, h_U) = {} & [\mu h_I + (1-\mu) h_U (0.5)] \ln [1 - \mu + 2\mu h_I / h_U] \\ & + [\mu(1-h_I) + 0.5(1-\mu)(1-h_U)] \ln [1 - \mu + 2\mu(1-h_I)/(1-h_U)] \\ & - 0.5(1+\mu)\ln(1+\mu) \end{aligned} \quad (12)$$

For $h_I = h_U$, $L(h_I, h_U) = 0$, so that $r(L) = r(H)$; this is likely only when $h_I = h_U = 1$, that is when short sales are allowed. We can show that, for any function of the form $F(h_I) = (Ah_I + B)\ln(C + Dh_I)$ we have differentiating twice, $d^2F/dh_I^2 = 2ACD/(C + Dh_I)^2$. Now consider $L(h_I, h_U)$ as a function of h_I for any given (μ, h_U), in the following form:

$L(h_I, h_U) = F_1(h_I) + F_2(h_I) - k$, where $F_1(h_I)$ and $F_2(h_I)$ are the first two terms of (12), and k is constant. Then both $F_1(h_I)$ and $F_2(h_I)$ fit the definition of $F(h_I)$ above, with:

$F_1(h_I)$: $A = \mu$, $B = (1-\mu)h_U/2$, $C = (1-\mu)$, $D = 2\mu/h_U$.

$F_2(h_I)$: $A = -\mu$, $B = \mu + (1-\mu)(1-h_U)/2$, $C = 1 - \mu + 2\mu/(1-h_U)$, $D = -2\mu/(1-h_U)$.

Substituting, both $d^2F_1/dh_I^2 > 0$ and $d^2F_2/dh_I^2 > 0$; hence, as the sum of two convex functions, $L(h_I, h_U)$ is convex in h_I, implying that it is minimised when $\partial L/\partial h_I = 0$. Then $\partial L/\partial h_I = 0$ when $h_I = h_U$, where, as we have seen, $L(h_I, h_U) = 0$. Thus, $I_P^L(P^0) = I_P^H(P^0)$ is the minimum of $L(h_I, h_U)$, implying that $I_P^L(P^0) > I_P^H(P^0)$ when $h_I \neq h_U$. **QED.**

Proof of Proposition 5: As in Proposition 3, $I_P^L(P^O)$ is the rate of convergence for all values of h_I

$$\begin{aligned} &= P^L(S)\ln\left(\frac{P^L(S)}{\gamma h_U}\right)+P^L(B)\ln(1-\mu)+P^L(N)\ln\left(\frac{P^L(N)}{\gamma(1-h_U)}\right) \\ &= [\mu h_I + (1-\mu)\gamma h_U]\ln\left(\frac{\mu h_I + (1-\mu)\gamma h_U}{\gamma h_U}\right)+P^L(B)\ln(1-\mu)+ \\ &\quad [\mu(1-h_I)+(1-\mu)\gamma(1-h_U)]\ln\left(\frac{\mu(1-h_I)+(1-\mu)\gamma(1-h_U)}{\gamma(1-h_U)}\right) \end{aligned} \quad (13)$$

and h_U. We have then:

$$F(h_I,h_U) = I_P{}^L(P^O)$$

We deal with the special cases first. At $h_I=h_U=0$,

$$\begin{aligned} F(h_I,h_U) &= (1-\mu)\gamma \ln(1-\mu) + P^L(B) \\ &= (1-\mu)\gamma \ln(1-\mu) + (1-\mu)(1-\gamma) \\ &= (1-\mu)\,[1+(\ln(1-\mu)-1)\gamma] \end{aligned}$$

At $h_I=0$, $h_U>0$,

$$\begin{aligned} F(h_I,h_U) = (1-m)gh_U \ln(1-m) + P^L(B)\ln(1-m) + \\ [m+(1-m)g(1-h_U)]\ln\left[\frac{m+(1-m)g(1-h_U)}{g(1-h_U)}\right] \end{aligned}$$

$$\frac{\partial F}{\partial h_U} = (1-m)g\ln(1-m) + 0 - (1-m)g\ln\left[\frac{m+(1-m)g(1-h_U)}{g(1-h_U)}\right] - (1-m)g + \frac{m+(1-m)g(1-h_U)}{(1-h_U)}$$

$$= (1-m)g\left\{\ln\left[\frac{(1-m)g(1-h_U)}{m+(1-m)g(1-h_U)}\right] + \frac{m}{(1-m)g(1-h_U)}\right\}$$

A minimum occurs at the solution to $\partial F/\partial h_U = 0$, which is equivalent to

$$\begin{aligned} \frac{\mu}{(1-\mu)\gamma(1-h_U)} &= \ln\left[\frac{\mu+(1-\mu)\gamma(1-h_U)}{(1-\mu)\gamma(1-h_U)}\right] \\ &= \ln\left[1+\frac{\mu}{(1-\mu)\gamma(1-h_U)}\right] \end{aligned}$$

This is of the form, $x = \ln(1+x)$, which has the unique solution $x = 0$, corresponding to $\mu = 0$; for $\mu > 0$, $\partial F/\partial h_U = x - \ln(1+x) > 0$. Hence $F(h_I,h_U)$ is increasing in h_U.

Furthermore, if for convenience we replace $\mu / [(1-\mu)\gamma]\ ]$ by k, we can express

$$\frac{\partial F}{\partial h_U} = \frac{1}{k(1-h_U)} - \ln\left[1+\frac{1}{k(1-h_U)}\right]$$

$$\frac{\partial^2 F}{\partial h_U{}^2} = \frac{1}{k(1-h_U)^2} - \frac{1}{k(1-h_U)^2+(1-h_U)} > 0$$

Hence, $F(h_I,h_U)$ is convex, increasing in h_U for $h_I = 0$ and $h_U < 1$.

At $h_I = 0$, $h_U = 1$, $F(h_I,h_U)$ explodes, since the absence of short sales by the uninformed means N is derived from insiders who wish to sell; hence, this is fully revealing. Along $h_U = 1$, for $h_I < 1$ the same result is true, until $h_I = h_U = 1$. Similarly, along $h_U = 0$, for $h_I > 0$ $F(h_I,h_U)$ explodes, since the lack of stock to be sold by the uninformed means S is derived from insiders; hence this also is fully revealing. (Thus $h_U=1$, $h_I<1$ and $h_U=0$, $h_I>0$ are fully revealing cases.)

At $h_I = 1 - (1-h_U)\gamma$, for $h_U > 0$, where the h_I boundary is derived from the standard condition that $1-h_I > (1-h_U)\gamma$,

$$F(h_I, h_U) = [m(1-g) + g h_U] \ln\left[\frac{m(1-g) + g h_U}{g h_U}\right] + P^L(B)\ln(1-m)$$

$$\frac{\partial F}{\partial h_U} = g\left\{\frac{-m(1-g)}{g h_U} + \ln\left[\frac{m(1-g)}{g h_U} + 1\right]\right\}$$

$\partial F/\partial h_U < 0$ since $\ln(1+x) < x$ and equals zero iff $\mu = 0$ or $\gamma = 1$. Hence $F(h_I, h_U)$ decreases along the boundary to $h_I = h_U = 1$.

From the expression (1) for $F(h_I,h_U)$, we have the partial derivative with respect to h_I:

$$\begin{aligned}\frac{\partial F}{\partial h_I} &= m\ln\left[\frac{m h_I (1-m) g h_U}{(1-h_U)}\right] - m\ln\left[\frac{m(1-h_I) + (1-m) g (1-h_U)}{h_U}\right] \\ &= m\ln\left[(m h_I + (1-m) g h_U)(1-h_U)\right] \; [m(1-h_I) + (1-m) g (1-h_U)]\, h_U \\ &= 0 \; iff \; h_I = h_U\end{aligned}$$

Furthermore, $F(h_I,h_U)$ is convex in h_I since

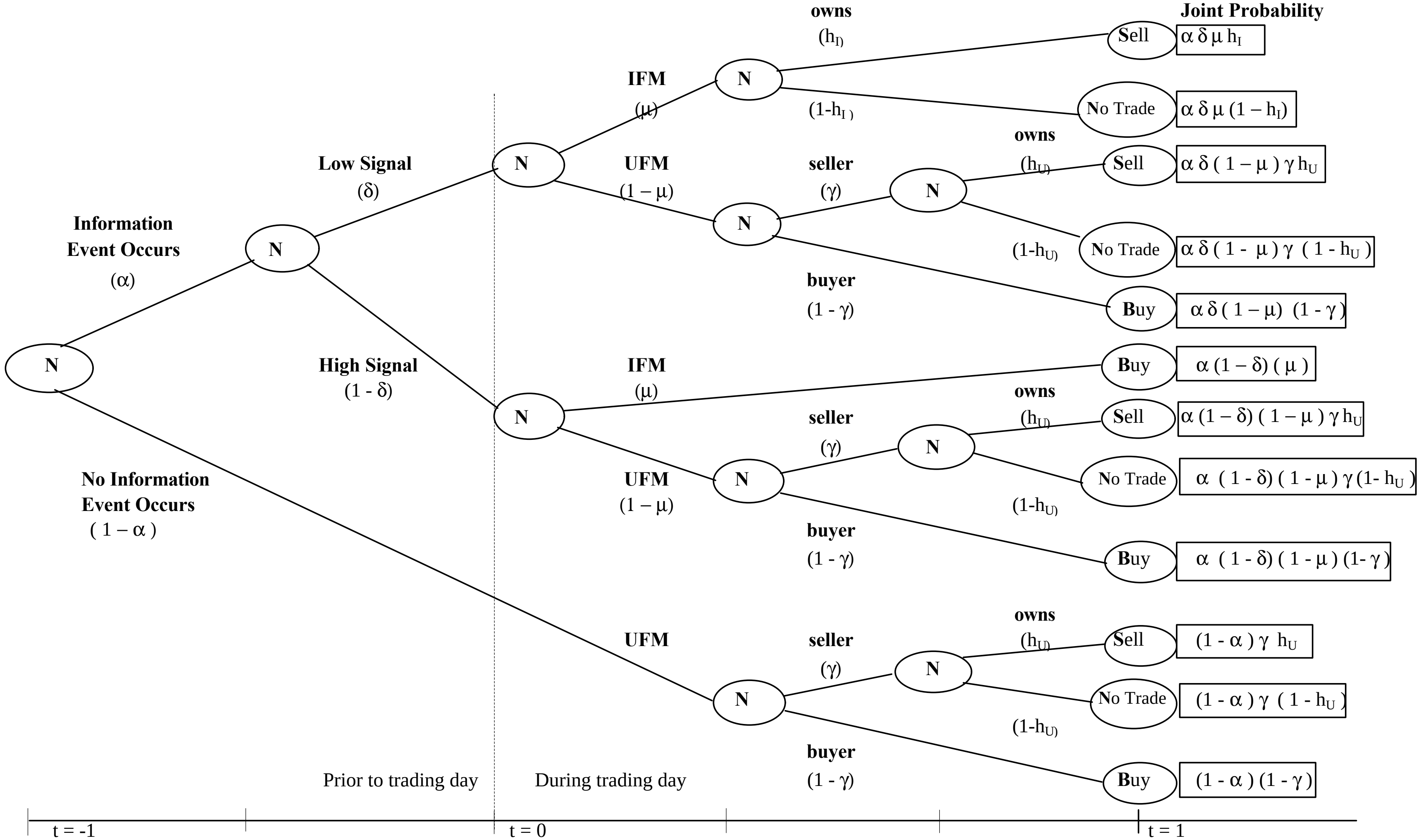
$$\frac{\partial^2 F}{\partial {h_I}^2} = m^2\left[\frac{1}{P^L(S)} + \frac{1}{P^L(N)}\right] > 0$$

Hence the minimum value for $F(h_I,h_U)$ (i.e. the minimum rate of convergence) for any h_U occurs along the line $h_I = h_U$. In fact, at $h_I = h_U$ the value of $F(h_I,h_U)$ reduces to a function of μ and γ:

$$F(h_I,h_U) = (\mu + (1-\mu)\gamma) \ln [(1-\mu) + \mu/\gamma]$$

Hence the value is constant and equal to the global minimum along $h_I = h_U$. **Q.E.D**.

Figure 1: Tree Diagram of The Trading Process



α = The probability that an information event occurs
δ = The probability of a low signal
μ = The probability that informed trader is selected
γ = The probability that an uninformed trader wants to sell

N = Event node, IFM = Informed insider trader, UFM = Uninformed liquidity trader
h_I = Probability that a IFM trader already owns the stock
h_U = Probability that a UFM trader already owns the stock
Nodes to the left of the dotted line occur at the beginning of the day
Nodes to the right are possible at each trading interval

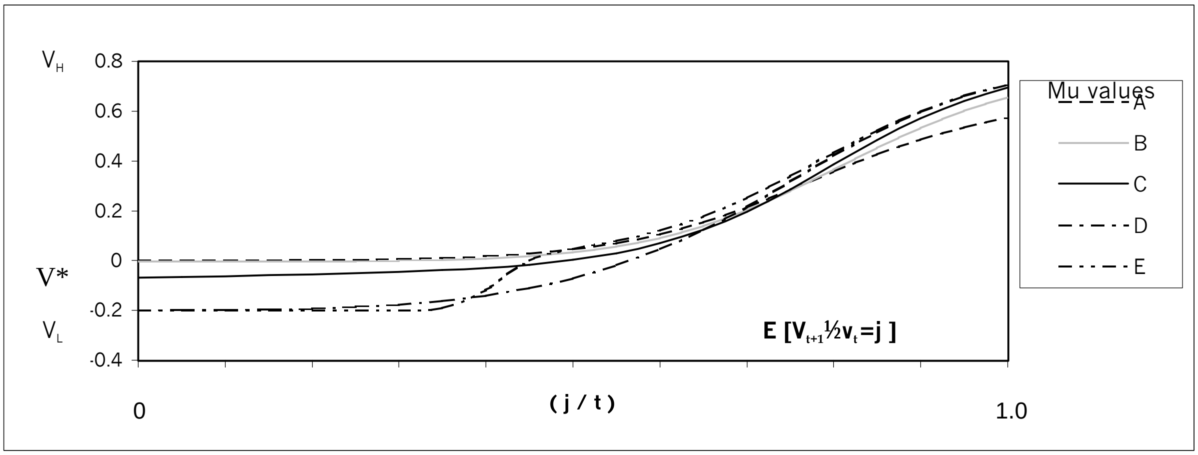
Figure 2: Price Revision Sensitivity to Volume of Trading in Theorem 1.

The figures give the conditional expected value at t+1 as a function of the trading intensity (j/t), for values of mu rising from .1 to .9; the conditional expected value for an assumed V* = 0 is given by

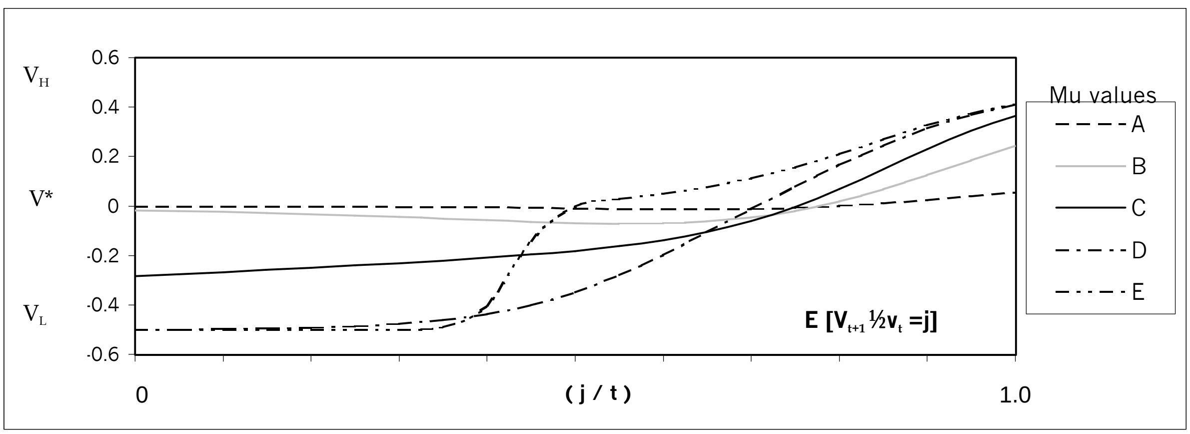
$$(V_H - V_L) \Pr\{ H | v_t \} * [(1 - \delta) - \delta \Pr\{ L | v_t \} / \Pr\{ H | v_t \}]$$

Figures 2a and 2b are rising, while Figure 2c, with delta > .5, has a varied pattern.

2a) $\delta = 0.2$



2b) $\delta = 0.5$



2c) $\delta = 0.8$

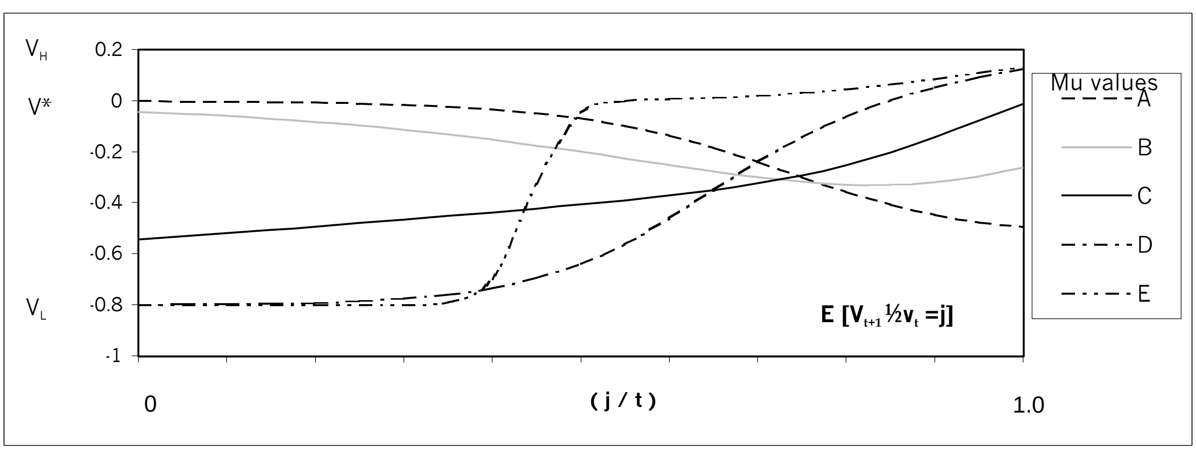


Figure 3: The Minimum Rate of Convergence Given a Low Signal Versus the Probability that the Selected Trader is Informed (mu). The convergence rate is increasing in both mu and the proportion of uninformed sellers (gamma).

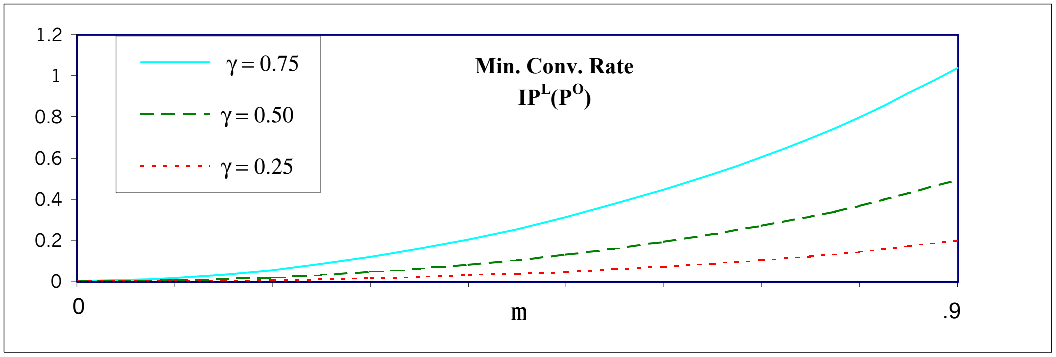


Figure 4a: The Minimum Rate of Convergence Given a High Signal Versus the Probability that the Selected Trader is Informed (mu). The convergence rate is increasing in both mu and the proportion of uninformed sellers (gamma).

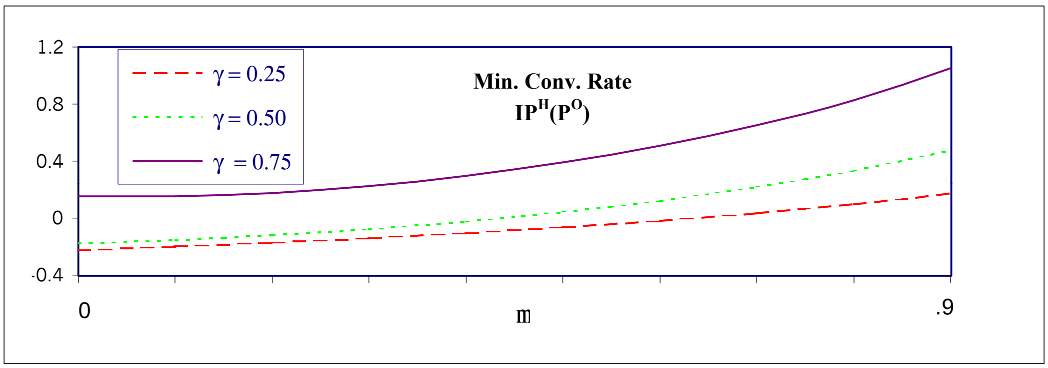


Figure 4b: The Minimum Rate of Convergence Given a High Signal Versus the Proportion of uninformed sellers (gamma). The convergence rate is increasing in the proportion of uninformed sellers (gamma), for all levels of ownership by uninformed investors.

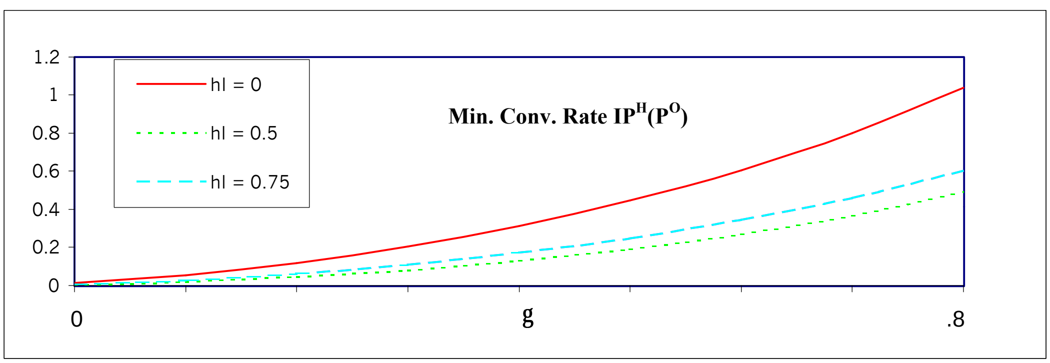


Figure 5a: The Minimum Convergence Rate Difference Between Low and High Signals versus the Probability that an Uninformed Trader Wants to Sell.

The difference $I_P^L(P^0) - I_P^H(P^0)$ is positive (faster convergence for low signal), for all values of gamma < .5 (majority of uninformed buyers) when the proportion of uninformed and informed traders is equal ($\mu = 0.5$).

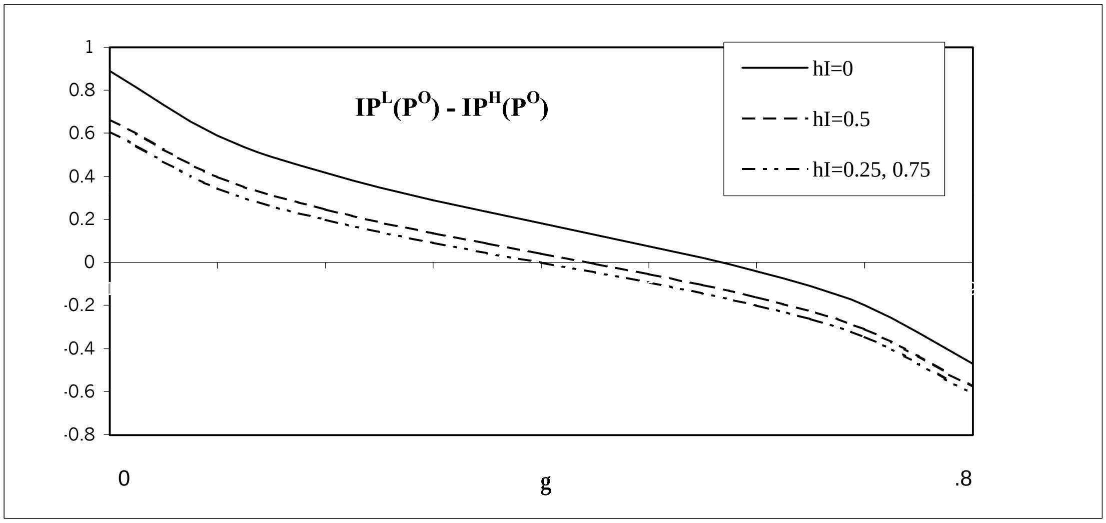


Figure 5b: The Minimum Convergence Rate Difference Between Low and High Signals versus the Probability that the Selected Trader is Informed.

The difference $I_P^L(P^0) - I_P^H(P^0)$ is positive (faster convergence for low signal), for all proportions of uninformed and informed traders (μ), when the informed. ownership (hI) is 0 or .5, and virtually identical at.25 or .75 ($\gamma = 0.5$).

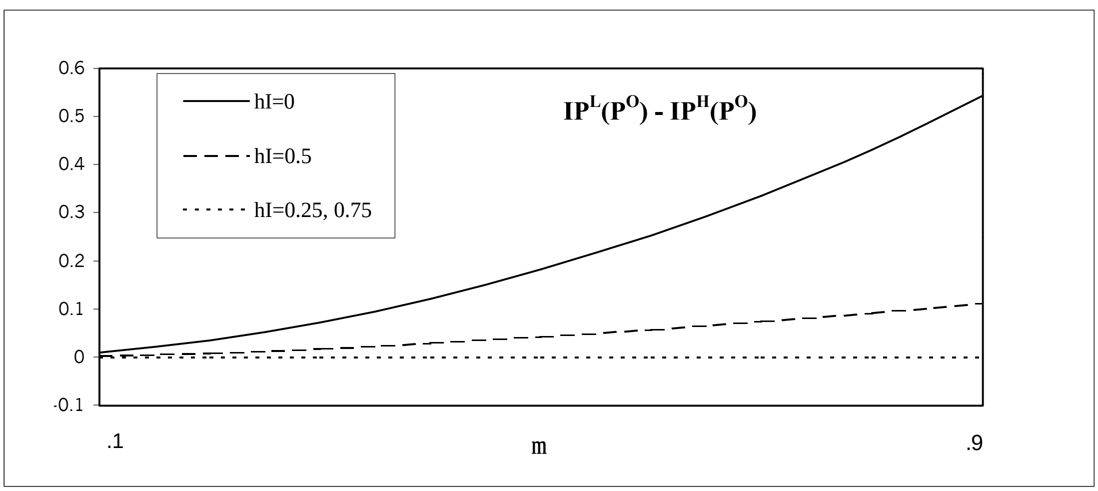


Figure 6a: Returns by decile for single-day trading. (The average single-day return for stocks whose previous day volume (T) lies within the global volume deciles.)

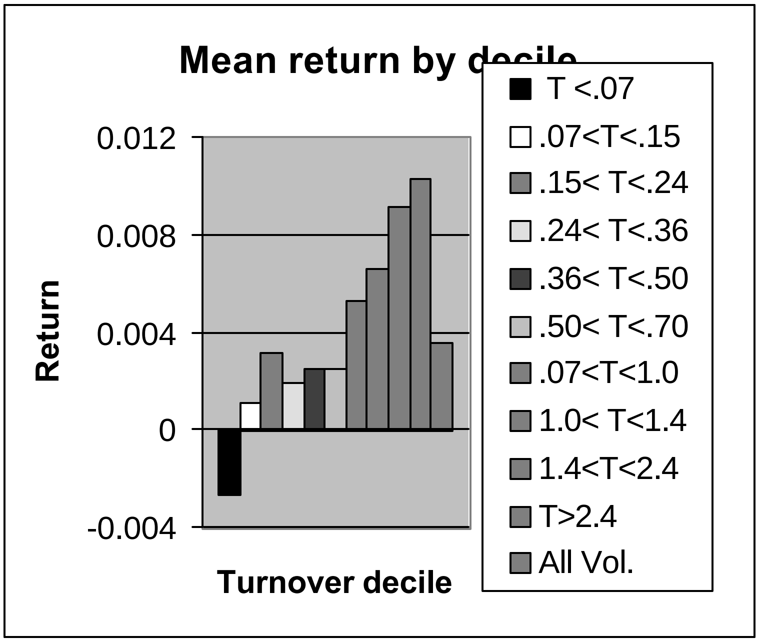


Figure 6b: Cumulative returns by decile intervals of single-day trading. (The cumulative average single-day return for stocks whose previous day volume (T) is less than the global volume deciles points

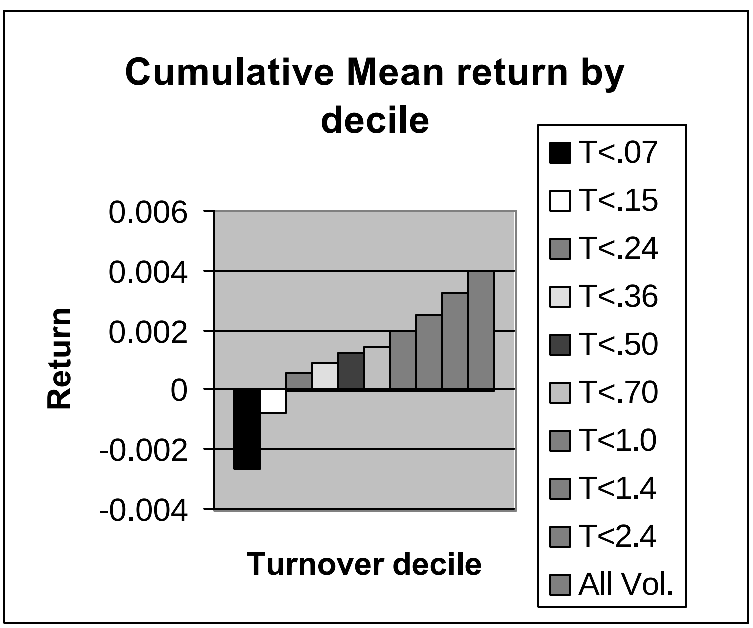


Figure 7a: Simulated value of the top decile volume portfolio versus the null strategies of buy-and-hold or re-balanced equal weight portfolios; an initial value of $60,000 is invested for the period Jan.16, 1998 - Dec. 31, 1999 in SET stocks.

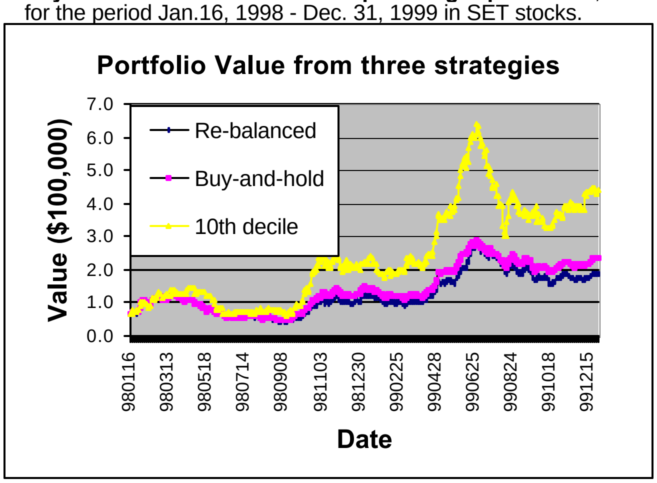


Figure 7b: Simulated value of the top decile volume portfolio net of that of the null strategies of buy-and-hold or re-balanced equal weight portfolios; an initial value of $60,000 is invested for the period Jan.16, 1998 - Dec. 31, 1999 in SET stocks.