Balance Sheet Effects on Option Pricing

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Abstract

A balance sheet structure including fixed assets, net working capital and risky long-term debt leads to a model for option pricing of the firm's equity. Each of the financial components constitutes a source of risk. A hedge based on three distinct options and the stock enables risk neutral valuation and avoids the problems of lack of tradability of the assets and market incompleteness reflected by the market price of risk. A two-option hedge when debt and working capital are combined leads to a unique partial differential equation yielding the call price. The combination of fixed volatility components exhibits non-constancy in stock return volatility, appearing to be stochastic volatility.

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The application of the Black-Scholes formula has focused on the valuation of ordinary stock options of a publicly-traded firm, usually restricted to call options. In their original paper, Black and Scholes (1973) referred to the establishment of a risk-neutral trading hedge as a solution to the discounting issue raised in previous models; yet they also devoted attention to the valuation of other instruments such as warrants, puts and the debt and equity of a leveraged firm. The latter approach was expanded by Geske (1979), who created a compound option structure for options on leveraged equity. This issue was extended by Rubinstein (1983) in an examination of the balance sheet combining a risky set of assets with both a non-risky asset and riskless debt in conjunction with the equity.

In the discussion of his resultant model, which he described as a displaced diffusion process, Rubinstein opened the door to a more thorough determination of the effect of the corporate financial structure on option pricing. For various reasons, he restricted his analysis to the case of a second non-risky asset, in place of the low-risk asset that he acknowledged to be a more accurate description, and to riskless debt, as well as incorporating known dividend payments. The result was a displacement of the diffusion process governing the risky asset by an amount that reflected the debt and non-risky asset proportions. He also noted the induced stochasticity for the volatility for the stock process, an area that has received considerable attention.

We re-consider the balance sheet analysis for the case of two or more risky components, thereby escaping Rubinstein's restriction to a single risky asset and riskless debt; he had imposed these modifications in order to avoid the non-tradability of the underlying sources of risk. Capturing the effects of the balance sheet structure on the return process of the traded equity helps to explain the phenomenon now known as the "volatility smile," and foreseen already by Black and Scholes, who referred to the bias in their seminal paper; Black also later described their work as being an initial step. Rubinstein addressed the bias in numerical demonstrations consistent with varying volatility, noting that his model appeared to "move option values in the right directions, relative to the Black-Scholes formula."

The formulation also creates a non-constant volatility, which description is more accurate than "stochastic volatility." The latter term implies that volatility has an actual, and presumably economically derived, stochasticity in its own right. Such a view has motivated a large body of analysis addressing the apparent statistical property of changes in volatility of the stock process, reflected by estimation of the Black-Scholes implied volatility; yet these approaches have not offered any justification for the existence of stochastic volatility. Such analyses usually result in an appeal to the "market price of risk," which in turn implies an incomplete markets framework. Our direction suggests that stochastic behaviour may be simply an apparition resulting from the combination in the balance sheet of individual component processes having fixed volatility.

In the next section, we discuss the Rubinstein assumptions and contrast them with ours. Then we describe the balance sheet model and develop the partial differential equation governing the option price. We identify alternative special cases for the risky assets being independent of risky debt and working capital; from these we produce two formulas for the option price, depending on which of the debt or working capital is larger. We also present demonstrations of the model's ability to fit market data. The appendix presents proofs of the theorems leading to a unique partial differential equation.

1. Rubinstein's Displaced Diffusion Process

Rubinstein established his model by analogy to the Black-Scholes process of creating a risk-free hedge based on the stochastic value of the stock return, which was a simple construct for a single risky asset. He discussed a model of the corporate structure with risky debt and two risky assets, but removed the risk from the low-risk asset noting that "it requires a more complex arbitrage argument and both portions of the firm need to be separately tradable." (p. 215) Rubinstein acknowledged that this modification was a distortion of reality "for firms that have two types of assets: those that are *relatively* risky and those that are *relatively* riskless." (p. 215) This characterization of real firms is particularly useful in describing a firm with future investment opportunities (high risk) as well as assets in place (low risk).

Our solution to this issue develops that "more complex arbitrage" using distinct options on the traded equity with alternative exercise prices. We then develop a partial differential equation (PDE) for the two risky asset and risky debt processes given the existence of three (or more) traded options on the same stock. A simplified differential equation, but similar in structure, follows when the returns for risky debt and the low risk asset are dependent; the combined asset, with a single underlying process, exists in the balance sheet as either net debt when debt exceeds net working capital, or net financial assets in the opposite case. This case leads to the familiar partial differential equation (PDE) for two asset processes results that is derived from alternative models, and only two distinct options are needed to hedge this case. Both PDEs are uniquely valid, even though the two risky assets are not traded; nor is there a need to introduce the market price of risk, as for instance in the Hull and White (1987) model for stochastic volatility. Hence our solution is a complete markets result.

The limited liability of the equity is a prime issue in modelling the balance sheet. Our balance sheet equation implies that limited liability does not exist, for debt that is not risky with respect to the asset value of the firm. There are two solutions to this problem, one being to model risky debt, as is explored here; the alternative is to adopt the compound option structure of Geske with riskless debt. Since the second approach involves a compound process based on two sources of asset risk to ensure limited liability, this would be a major complication. It is, however, unnecessary by Rubinstein's argument, "Although the debt is not risky, the assumption of riskless debt is reasonable for most firms with listed options, since preliminary analysis of the compound option model shows that the levels of debt risk usually present are sufficiently low to have negligible impact on option values." (p. 215) Thus, on an empirical basis, he rejects the need for a valuation model to include the compound option model. In fact, the requirement of limited liability is also obviated by an option exercise price well above the point where probability mass would materially affect its pricing.[1]

It should be recognized that Rubinstein is motivated by the wish to avoid two sources of risk when he claims that risky debt is an unnecessary feature of the model, after arguing against a second risky asset, in order to construct the usual Black-Scholes hedge. Because of our use of additional options in creating the hedge, risky debt does not pose a problem; in fact, the recognition of the debt factor leads to greater opportunities to produce more complex option functions.

With all other factors being observable or defined, the Black-Scholes accuracy issue revolves around the *formulation* of the volatility of the stock return process. Many models have modelled the perceived stochastic volatility, most notably Hull and White, and used this to address the observed bias across exercise prices. Empirical evidence suggests that none of these models is any more effective than the Black-Scholes model in fitting observed data and eliminating the smile.[2] Nor is any model justified by an explanation for the existence of the perceived volatility of variance. Rather, having observed from the implied volatility that there was an apparent volatility of the stock process variance, financial researchers have used statistical methodology to model that variance and then determined its effect on the resulting option price. At the other extreme, the CEV class maintains a fixed volatility while appplying the variance parameter in varying ways; this too appears not to be successful in explaining pricing biases.

These approaches provide alternative descriptions of the stock return process and its volatility which lead to variations in the modelled price. The theory and results, therefore, are statistical analyses of the pricing bias, rather than economic models with simpler return processes as proposed in our model and earlier by Rubinstein. His discussion of relatively high or low risk assets implicitly refutes the approximation of the return process as being derived from a single homogeneous portfolio of assets with a lognormal distribution. (The appeal of the lognormal distribution lies in its mathematical tractability and reasonable fit to observed stock returns over longer periods.) A more complicated asset structure involving distinct assets and debt retains the lognormal benefits but yields a return process for the equity that is not lognormal.

Consider the following scenario in which two firms (A and B) with simple structures exist, both having optioned equity. With each having a homogeneous asset structure and no debt, assume the equity of each follows a lognormal process (probably of higher and lower variance), so that the Black-Scholes model applies to both. Now assume that the two firms merge; firm A buys firm B by issuing debt. The resulting firm AB now has debt in its structure and its assets are two independent entities, each following a distinct lognormal process. Options are issued on the firm AB following the merger. Can one reasonably expect the new AB equity options to be valued on the basis of the Black-Scholes model? Would one expect the value of the AB options to be the sum of the options on A and B prior to the merger, or any simple modification of the individual values? Or would they require a model such as we have proposed?

We have focused on the identification of the two classes of assets as fixed and current as a convenient interpretation of the balance sheet that fits the high- and low-risk characterization; for example, "fixed" assets, including some intangible assets like intellectual capital, produce cash flows of quite uncertain outcome, whereas current assets have a relatively low and less variable rate of return, barring non-payment and inventory write-downs. We actually identify the low-risk asset as net working capital, leaving only long-term debt and equity on the right side of the balance sheet.

Rubinstein discussed the effects on the option value induced by changes in the debt ratio and the riskless to total asset ratio; the results were far more complicated than monotonic responses and were productive in explaining observed bias. Although he describes the volatility of the firm value as not being constant, but rather stochastic, he does not argue that there is some cause other than the corporate structure for the apparent "stochastic" behaviour.

Rubinstein further argues against the modelling of two risky assets by noting the greater requirements for parameter estimation, as well as the need for numerical integration that would follow the assumption. In fact, more parameter flexibility is perhaps an advantage for modelling and testing, but essentially these are technical rather than theoretical considerations. In our model, assuming the independence of the returns of high-risk assets from both debt and net working capital allows us to solve the two-factor PDE for a closed form expression for the option value requiring, indeed, numerical integration. Our expression for the option value is based on parameters for the proportion of long-term debt and for the initial ratio of the high-risk to total assets, as well as the individual variances of risky components, in addition to the standard parameters of exercise price, riskless rate and time to maturity. Essentially, this has increased the number of unobserved parameters (the variances) from one to two; however, in the context of an empirical investigation, the parameters for the relative low- and high-risk asset mix and the level of debt can be taken as unknown. The resulting four parameters can be manipulated to determine whether option prices consistent with a smile for a single variance can be explained by a fixed set of the four parameters. We believe that the introduction of the parameters of the corporate structure provides a more promising explanation for the smile phenomenon.

II. The Balance Sheet Model

The assets of the firm are segregated into fixed assets, having typical business risk, and net working capital (current assets minus current liabilities), having relatively low risk.

Financial leverage is then incorporated by introducing risky long-term debt, whose market value changes over the expiry period of the option. The standard balance sheet relationship then follows:

$$U(t) + V(t) = L(t) + S(t)\,, \tag{1}$$

where $U(t)$ = market value of fixed assets (per share)

$V(t)$ = market value of net working capital (per share)

$L(t)$ = market value of long-term debt (per share)

$S(t)$ = market value of equity (per share)

The resulting stochastic process for the stock price is then:

$$dS(t) = dU(t) + dV(t) - dL(t)\,. \tag{2}$$

For simplicity, we assume both assets and debt to follow the usual lognormal processes, although this restriction is not essential, so that:

$$dU(t) = \mu_1 U(t)dt + \sigma_1 U(t)dz_1\,, \tag{3}$$

$$dV(t) = \mu_2 V(t)dt + \sigma_2 V(t)dz_2\,. \tag{4}$$

$$dL(t) = \mu_3 L(t)dt + \sigma_3 L(t)dz_3\,. \tag{5}$$

Noting then that the traded equity upon which the option is written is a function of the two assets and debt, the conventional formula for a European call option at time t, with strike price X and expiration time T is re-expressed as:

$$C[S(t), t, X, T] = C[U(t), V(t), L(t), t; X, T]\,, \tag{6}$$

Differentiation of the right-hand-side expression yields separate partial derivatives with respect to both risky assets and the debt. For simplicity, we suppress the time index t for initial time $t = 0$ and refer to:

$$C[U, V, L; X, T; r, \sigma_1, \sigma_2, \sigma_3]\,. \tag{7}$$

Given the existence of several traded options on the same stock with varying strike prices and maturities, it is possible to hedge the three underlying sources of risk by a portfolio with three call options held long and the underlying stock held short:

$$P = \alpha C^{A} + \beta C^{B} + \gamma C^{G} - S\,, \tag{8}$$

where A, B and G denote three arbitrary call options on the same stock, with $\alpha = \alpha(\mathrm{t})$, $\beta = \beta(\mathrm{t})$ and $\gamma = \gamma(t)$ being their instantaneous weights, respectively.

Lemma – The hedge portfolio (α, β, γ) is independent of the underlying processes, and earns a riskless rate of return, if the following set of equations holds:

$$\alpha C_{U}^{A} + \beta C_{U}^{B} + \gamma C_{U}^{G} = 1 \tag{9}$$

$$\alpha C_{V}^{A} + \beta C_{V}^{B} + \gamma C_{V}^{G} = 1 \tag{10}$$

$$\alpha C_{L}^{A} + \beta C_{L}^{B} + \gamma C_{L}^{G} = 1 \tag{11}$$

$$\alpha F(A) + \beta F(B) + \gamma F(G) = -r(U + V - L) \tag{12}$$

where

$$\begin{aligned} F(A) = C_{t}^{A} - rC + \tfrac{1}{2}(\sigma_{1}^{2}U^{2}C_{UU}^{A} + \sigma_{2}^{2}V^{2}C_{VV}^{A} + \sigma_{3}^{2}L^{2}C_{LL}^{A}) \\ + (\rho_{12}\sigma_{1}\sigma_{2}UVC_{UV}^{A} + \rho_{13}\sigma_{1}\sigma_{3}ULC_{UL}^{A} + \rho_{23}\sigma_{2}\sigma_{3}VLC_{VL}^{A}) \end{aligned} \tag{13}$$

where C_{x} denotes the partial derivative of C w.r.t. x, with *F(B)* and *F(G)* defined similarly for options *B* and *C*.

Proof: See the appendix.

Theorem – Provided there exist at least three traded options on the same stock, the equilibrium price of an option on the equity of a firm with two risky assets and risky debt must satisfy the following PDE, subject to relevant boundary conditions.

$$\begin{aligned} &C_t + rUC_U + rVC_V + rLC_L + \tfrac{1}{2}(\sigma_1^2 U^2 C_{UU} + \sigma_2^2 V^2 C_{VV} + \sigma_3^2 L^2 C_{LL}) \\ &+ (\rho_{12}\sigma_1\sigma_2 UVC_{UV} + \rho_{13}\sigma_1\sigma_3 ULC_{UL} + \rho_{23}\sigma_2\sigma_3 VLC_{VL}) - rC = 0 \end{aligned} \tag{14}$$

Proof: See the appendix for the proof of the theorem. The assumption of a lognormal process is irrelevant to the proof, although a jump process is precluded.

Observability and tradability of assets

The asset components of the balance sheet structure are clearly not traded as financial instruments. On the other hand, the value of the low risk assets, if described as net working capital, is observable from the corporate balance sheet in book value. Securitization of assets such as inventories and receivables also affords an opportunity to establish a market value for the working capital. The balance sheet equation (1) then permits the implicit determination of the high risk asset value *(V(t)= S(t) + L(t) – U(t))* without requiring the actual trading of *V(t)*. In modeling, we require that the balance sheet be specified by two parameters: the current proportion of fixed assets to total assets

$$a(t) = \frac{U(t)}{U(t) + V(t)} \tag{15}$$

and the debt/equity ratio

$$b(t) = \frac{L(t)}{S(t)} \tag{16}$$

These stochastic parameters, which correspond to Rubinstein's α and β, are assumed to be implicitly determinable at any time.

To clarify, the model requires the values and the distributional properties of assets *U(t)* and *V(t)*, but it does not require that these assets actually be traded as market instruments. In calculating a value for the option, as in formula (21) below, values of *U(t)* and *V(t)* must be found by observing for a firm the market value for *V(t)* and asset ratio *a(t)* and then inverting (15) to produce $U(t) = a(t)V(t)/[1-a(t)]$. In an empirical examination, it must be assumed that the market implicitly estimates the parameters in setting a price for options on the basis of the model. The option value for various assumptions of *a(t)* and *b(t)*, which would represent differently structured firms, will exhibit various patterns for different exercise prices, thereby producing the smile characteristics.

A solution for uncorrelated asset processes

Standard numerical procedures would produce a solution to the PDE (14). Note that the correlation coefficients ρ_{ij} offer additional degrees of freedom in determining the option price, but greatly increase the complexity of solving the PDE. In order to obtain a closed form expression for the option value, however, it would be necessary to assume zero correlation between the three processes, leading to a triple integral expression. The presumed independence of all three components, however, is not realistic. Since there is little immediate connection between fixed assets, whose value depends upon product markets, and debt trading in debt markets, it is reasonable to assume that their correlation is negligible. On the other hand, the correlation between short-term financial assets and long-term debt, the subject of much analysis in the term structure field, is high. We assume it to be relatively perfect over a short time period before option expiry. Both of these assumptions are clearly false in the extreme, but are needed in order to generate a simple solution.

In the case of reducing the three independent components to two, only two options would be required to create the riskless hedge. This would simplify the development presented in the Lemma and Theorem and lead to a simplification of equation (14):

$$C_t + rUC_U + rDC_D + \tfrac{1}{2}(\sigma_1^2 U^2 C_{UU} + \sigma_4^2 D^2 C_{DD}) + \rho\sigma_1\sigma_4 UDC_{UD} - rC = 0 \qquad (17)$$

The PDE (17) is recognizable as the well-known equation for the case of two tradable assets; what is remarkable is that it also applies here even though the assets are not assumed to be tradable, other than implicitly through equity values.

If there exist two independent processes, the conditional distribution of the stock price can be expressed as a convolution of two distributions. The fixed asset process is naturally described as lognormal, but the debt process is not necessarily so. The distribution of debt is affected by various factors, including the restriction that the maximum long-term debt value is the undiscounted sum of future interest and principal, or the face value if pure discount debt. As well, it is a reasonable presumption that market value changes due to interest rate risk are minimal in a single period, while changes due to credit risk (deriving from the distribution of asset value) have a maximum range of zero to face value plus one period interest. Finally, the distribution is likely to be fairly normal or lognormal with very low coefficient of variation, so that the difference between the two is inconsequential. For that reason, it is convenient to accept lognormality for consistency with other models.

In the balance sheet we retain the two financial components as separate values, but we determine the return on one as simply a multiple of the return on the other. In the case of *L(t)- V(t)*>0, we can define *N = (L-V)/V*, as the initial ratio of net financial liabilities to net working capital. We then substitute for L_T, using only the distribution $h(V_T)$. This implies that L_T - V_T must remain positive throughout the period.

For two uncorrelated processes, the conditional distribution of the stock price can be expressed as a convolution of two lognormal distributions:

$$f(S_T, T; S, 0) = \int_0^\infty g(S_T + L_T - V_T) * h(V_T) dV_T \,, \tag{18}$$

where

$$\begin{aligned} g(U_T, T; U, 0) &= \frac{1}{U_T \sigma_1 \sqrt{2\pi T}} \exp\left[-\frac{\left[\ln U_T - \ln U - (r - \sigma_1^2/2)T\right]^2}{2\sigma_1^2 T} \right], \\ h(V_T, T; V, 0) &= \frac{1}{V_T \sigma_2 \sqrt{2\pi T}} \exp\left[-\frac{\left[\ln V_T - \ln V - (r - \sigma_2^2/2)T\right]^2}{2\sigma_2^2 T} \right]. \end{aligned} \tag{19}$$

It is clear that the stock price process governed by the conditional probability transition density (18) can be fairly complicated; certainly it cannot be approximated by any simple process with constant volatility parameter, despite the simplicity of the component processes giving rise to densities *g* and *h*.

As in Cox and Ross (1976), risk-neutral valuation gives the solution to (18):

$$C = e^{-rT} \int_{-\infty}^{\infty} \max(0, S_T - X) f(S_T, T; S, 0) dS_T \,. \tag{20}$$

We first define *W = (V - L)/V*, as the initial ratio of net financial assets to net working capital, and substitute for L_T while using the distribution $h(V_T)$. By the revised balance sheet equation, positive payoff region for the option, $S_T > X$, is equivalent to $U_T + V_T - L_T > X$ or $U_T > X - V_T W$. If *V(t) - L(t)*<0, then *W<0* and range of the outer integral (over V_T) would simply be (0, ∞), with the inner integral over ($X - WV_T$,∞), giving:

$$C(U,V,L,X,T;r,\sigma_1,\sigma_2)=e^{-rT}\int_0^\infty\int_0^\infty \max(0,U_T+WV_T-X)g(U_T)h(V_T)dU_T dV_T$$

$$=\int_0^\infty e^{-rT}\left\{\int_{\mathrm{X\text{-}WV_T}}^\infty (U_T+WV_T-X)g(U_T)dU_T\right\}h(V_T)dV_T$$

$$=\int_0^\infty \left[UN(d_1)-[(X-WV_T)e^{-rT}]N(d_2)\right]h(V_T)dV_T \qquad (21)$$

where

$$d_1=\frac{\ln[U/(X-WV_T)]+(r+\sigma_1^2/2)T}{\sigma_1\sqrt{T}},$$
$$d_2=d_1-\sigma_1\sqrt{T}, \qquad (22)$$

and $N(\cdot)$ denotes the normal distribution function. Notice that the integrand is the standard Black-Scholes formula with U being the asset price and $X+D_T$ being the exercise price, and the solution involves a simple numerical integration over V_T. Hence, we can re-express the solution as:

$$C(U,V,L,X,T;r,\sigma_1,\sigma_2)=\int_0^{X/W}\left[BS[U,(X-WV_T),\sigma_1]\right]h(V_T)dV_T \qquad (21a)$$

In the second case of *V(t)-L(t)*>0, the procedure is more involved due to the existence of a negative range for the inner integral arising from the positive financial assets. Since $U_T \geq 0$ and $V_T \geq 0$, the outer integral (over V_T) would be divided into two parts, first on the range (0, *X/W*), with the inner integral over ($X-WV_T$,∞), and then over (*X/W*,∞), with the inner integral over (0,∞), giving:

$$C(U,V,L,X,T;r,\sigma_1,\sigma_2)=e^{-rT}\int_0^\infty\int_0^\infty \max(0,U_T+W(V_T-X/W)g(U_T)h(V_T)dU_T dV_T$$

$$=\int_0^{X/W} e^{-rT}\left\{\int_{X-WV_T}^\infty [U_T-(X-WV_T)]g(U_T)dU_T\right\}h(V_T)dV_T$$

$$+e^{-rT}\int_{X/W}^\infty\left\{\int_0^\infty (U_T-(X-WV_T)g(U_T)dU_T\right\}h(V_T)dV_T$$

$$= \int_0^{X/W} \left[UN(d_1) - [(X - WV_T)e^{-rT}]N(d_2) \right] h(V_T) dV_T$$

$$+ (V - L)N(d_3) + [U - e^{-rT} X]N(d_4), \tag{23}$$

where

$$d_1 = \frac{\ln[U/(X - WV_T)] + (r + \sigma_1^2/2)T}{\sigma_1 \sqrt{T}},$$
$$d_2 = d_1 - \sigma_1 \sqrt{T},$$
$$d_3 = \frac{\ln[(V - L)/X] + (r + \sigma_2^2/2)T}{\sigma_2 \sqrt{T}}, \tag{24}$$
$$d_4 = d_3 - \sigma_2 \sqrt{T}$$

Notice that the integrand is the standard Black-Scholes formula with U being the asset price and $X - WV_T$ being the adjusted exercise price, while the second is also a Black-Scholes value with V-L as the asset price and the actual X as the exercise price, plus an added value from U. We can then re-express the solution more simply as:

$$C(U, V, L, X, T; r, \sigma_1, \sigma_2) = \int_0^{X/W} \left[BS[U, (X - WV_T), \sigma_1] \right] h(V_T) dV_T$$

$$+ BS[V - L, X, \sigma_2] + UN(d_4) \tag{23a}$$

with

$$d_4 = \frac{\ln[V/(X + e^{rT} L)] + (r - \sigma_2^2/2)T}{\sigma_2 \sqrt{T}} \tag{24a}$$

This solution is very similar to the solution of Hull and White (1987), which also assumes zero correlation between the stock price process and the volatility process; in their case, numerical integration is over the volatility range. Our formulation benefits, however, from the completeness of the market, compared to Hull and White's need to estimate a market price of risk for the non-tradable volatility. Note also that the solution reduces to the Rubinstein displaced diffusion model if $\sigma_2 = 0$ for low-risk *V(t)*, and to the Black-Scholes model if further, *L(t)* and *V(t)* are fixed at zero.

III. A Simulation and Data Fitting

In testing the hypothesized model, parameter estimation needs to be consistent with the underlying structure. The approach should be used for individual companies with their actual, although not perfectly observable, balance sheet structures. The two variance parameters would then be estimated from the option price observations. Alternatively, these two plus the *a(t)* and *b(t)* parameters can be varied for a best-fit solution, and the results compared with the actual balance sheet data. The model can also be used for index data, where the parameter estimates must be compared with actual estimates of the average capital structure of the component firms listed in the index; similarly, the asset ratio would have to be assessed for reasonableness against the ratios of component firms. Another test of validity of the model is stability of the estimates across time, especially of the estimated balance sheet parameters. While the variance estimates can be expected to vary in time, as for the conventional estimates of Black-Scholes implied volatility, the balance sheet should remain relatively stable. We have investigated whether the model could provide an explanation for the anomalous results known as the "volatility smile."

The Volatility Smile

Empirical studies of the Black-Scholes option pricing model and its variants have repeatedly revealed pricing biases that are inconsistent with the theoretical foundations of the model. The phenomenon of bias in the volatilities implied by market prices of options on the same asset, rather than a single volatility across the range of exercise prices (and maturities), is known as the volatility smile. Alternatives to the standard specification of the underlying diffusion process, including volatility corrections and jump processes or mixed diffusion-jump processes, have proven to be no more successful than the Black-Scholes model in matching market prices. Rubinstein (1985) tested alternatively specified models, which were unable to eliminate the smile. Subsequently, Rubinstein (1994) and Jackwerth-Rubinstein (1996) developed an alternative that they claimed to be successful in explaining the market's post 1987 smile. They have shown that the implied risk-neutral distribution revealed by the market prices can be linked to a binomial tree, thus preserving the property of risk-neutral valuation. As important as the theoretical analysis is the empirical finding that the implied distribution for the aggregate return is not lognormal; instead, the implied distribution exhibits both kurtosis and skewness, and frequently is bimodal, as may occur with mixtures of lognormals. They used a non-parametric approach to match results with the most general distribution needed to fit the data, but this required a major computational effort for the efficient recovery of the implied distribution from the data.

Setting our $\boldsymbol{\sigma_2} = 0$ for a single risky asset, reduces to Rubinstein's model, which further degenerate to a Black-Scholes value for **a**=1 and **b**=0. If the modeled call values are then used to create implied volatilities (IVs) by Black-Scholes, then cases of a high proportion of high-risk assets (**a** = 0.75) showed similar results to Rubinstein's (which exhibited a typical post '87 crash smile decreasing in the exercise price). The two models differ quite significantly when asset *U* counts for a smaller proportion (**a** = 0.25); steeper negatively-sloped smiles occur, in contrast to Rubinstein's flatter and even positivelysloped smiles. The effect of **b** (leverage ratio) was important in demonstrating the adaptability of the price to the parameter choice. For **b** = 0 (no leverage), I.V. was increasing in exercise price at **a** =0.75, but decreasing at **a** = 0.25; for **b** = 2 (high leverage), the smile reversed, decreasing at **a** =0.75 and increasing at **a** = 0.25. The added flexibility produced smiles that could fall in the exercise price or stay constant, while Rubinstein's was rising as **b** changed.

Estimating Parameters

In fitting parameters to observations, it was necessary to determine the more precise response across the exercise price spectrum to parameter changes. Although the response can be determined analytically for simple changes (for instance, the call value is obviously rising in the variance parameters and in the debt ratio), the magnitude of price changes to individual parameter variation and also in combinations causes an analytical approach to become fairly intractable. It is also noteworthy that the normal volatility smile for a single return variance (a two-dimensional graph) is replaced by a smile in two variance parameters, requiring a different method of illustration for the biases.

To examine the parameter effects, a few sampled data points from U.S. index options have been used. The process involved setting a representative parameter for the market dividend yield (initially 3%), then pre-selecting a starting value for the variance of the stock by inversion of the Black-Scholes formula for a given call price (at the applicable time to maturity, corresponding risk-free rate, index level and closest to-the-money exercise price). The total stock variance can be decomposed into independent σ_1 and σ_2 (whose weighted sum of squares equals the total variance given independence of the assets), with σ_2 = .035 as a reasonable estimate of low-risk volatility[3]; σ_1 is then determined from the equation

$$\sigma^2 = a^2\sigma_1^2 t + (1-a)^2\sigma_2^2 \qquad (25)$$

where σ was the implied volatility. (The null valuation corresponds to parameters a and b at values of one and zero, respectively.) Parameters **a** (fixed asset ratio) and **b** were then varied from .05 to .95 by increments of .10 and the modeled call price was estimated from expressions (23) and (24), modified to include a dividend stream. The immediate result was to detect the variation in the call price in response to the parameter variation. Since the initially calibrated value was for no debt and no low-risk assets, increasing the parameters naturally increased the observed call price, from 27.904 to the values presented in Table 1. The table also shows the value of σ_1 changing with parameter **a** to retain the total variance at the B-S implied variance level. The immediate significance of this is to demonstrate that the model produces differing call prices, even though the stock variance remains constant (as is predicted from the model).

The next step was to force the call price to remain constant at the initial (observed) value for the same set of variations in parameters **a** and **b**. This requires that overall variance and σ_1 change to compensate for increasing or decreasing leverage and the asset structure. The values of these are presented in Table 2A and 2B. So far, this merely confirms that the observed price can be produced by any of the discrete pairs of parameters (**a,b**), or any interpolated values; note that there remains freedom to adjust the low-risk parameter σ_2 from its initial value of .035, with corresponding (**a,b**) values. The test is then to calculate from the model the values of the calls with alternative exercise prices, using the same assumed values for the parameters, and to compare these with the observations. In this case, the starting call had **X**=425 and the second call had **X**=400. Table 3A presents the value of the second call, where the call price is seen to be decreasing in **b** for all **a**; the price is concave in **a** for all **b**. Table 3B presents call values for **X**=450, which are relatively convex in both parameters.

It is evident that the model gives clear opportunities to create smiles using actual data. After fitting **a** and **b** parameters to the first call, however, neither of the alternative calls were trading at values within the ranges produced, under the assumed values for σ_2 and the risk-free rate and dividend yield. These also can be varied, noting that the risk-free rate and dividend yield must be consistent with observable values. This negative initial finding is neither conclusive nor fatal. Relaxing trade prices to points within the bid-asked spreads dramatically increases flexibility. Furthermore, the preliminary approach was to choose intuitively appealing values for some parameters such as σ_2, but not necessarily correct ones.

The next stage of testing consisted of using the entire data set for the first time point (being one time window on a single day – January 4, 1993). The data consisted of 18 simultaneous exercise price options of 3-months (74 days), 11 of 6-months (165 days), 3 of 9-months (256 days), and 4 of 1-year (347 days); the exercise prices ranged from 375 to 460 (by intervals of 5 for the 3-month), with an index level of 436.96, and applicable interest rates rising from .032 to .035 across the maturities.

For this data set, the implied B-S volatility was calculated from the closest option at each maturity, being either 435 for the shorter options and 425 for the longer two. This volatility was then used to find the B-S price across the exercise spectrum, for each maturity (as a means of defining the pricing error for comparison with our modelled price error, in contrast to using a volatility smile as evidence of error). The smile phenomenon was apparent, as the one-year, four-option set at 375, 400, 425, 450 had increasing implied volatilities of .1205, .1206, .1247, .1274, respectively. These volatilities were also used as starting points for the parametric pricing. At this point, it became apparent that for both models, the pricing error was reduced if the assumed dividend yield was reduced, and subsequently a rate of $q = .01$ was used.

Due to the use of implied volatility as a starting point, the programming uses $\boldsymbol{\sigma}$ (overall) as a parameter and then determines $\boldsymbol{\sigma_1}$ from expression (25). Hence the effective parameter variation is in combinations of the $\boldsymbol{\sigma}$, **a** and **b** values. The first result of scanning the parameter ranges gave a fit of all four one-year options between their bid-asked ranges (but not *at* their mid-points); this success was greatly enhanced by finding that the parameter combination also fit the 9-month bid-asked ranges, even with the same $\boldsymbol{\sigma}$. Because these results were not consistent with the shorter and wider-ranging options, the eleven 6-month options were used to find the least biased set of parameters with respect to the market prices. Then these parameters were tried for the other three maturity sets. The results are portrayed in Figures 1-4, where the model and B-S pricing errors (model – market) are graphed across the exercise prices.

In Figure 1, the one-year bias is apparent in the B-S pricing, under-pricing the market by \$.60 in the money, and over-pricing by \$1.50 out of the money. (One should recognise the volatility smile equivalent, which would be negatively sloped.) The parametric model, on the other hand, produces a \$.50 to \$.25 under-pricing in the near-the-money region with \$2.30 over-pricing in-the-money. By the standard of fitting near the money it clearly prices better. Figure 2 shows both under- and over-pricing by the model but far less error than for the B-S model. Figures 3 and 4 each show U-shaped pricing errors for both models, with the parametric model giving an improved fit in each case, with far less bias towards the slope of the error.

Figure 4, reflecting the shortest and most actively traded option series, bears the closest examination. The fit is relatively close in the region surrounding the current price with the same bias as the B-S values, but returns to the market prices when in the money

The parametric model is less under- and over-priced, and essentially fits worse at-the-money where the B-S is fitted by implied volatility. Evidently, reducing the effects of a and b and using the implied volatility gives the B-S result. Increasing a and b and decreasing sigma together tends to raise the slope of the price, i.e., raising prices for high X and lowering them for low X. Generally, the model allows the positive error slope to be reduced and occasionally to be reversed. (This is equivalent to reducing the negatively sloped smile.)

Essentially the results are extremely encouraging, in that they show that the "best fit" can be obtained with a consistent parameter set. A better fit can be obtained by allowing the volatility to vary over the maturity; we chose to maintain all parameters at the same value to demonstrate the potential for consistent estimates. While a constant sigma is implied by the B-S lognormal distributional assumption (barring a GARCH or stochastic volatility model), it is rarely in practice observed, so a slight variation in sigma is tolerable. More crucial to the model is that the asset and liability structure not be open to much change, even over a period of successive days of observations. Hence the finding that the "best fit" was produced by exactly the same **a** and **b** parameters is significant. Also, the values used, debt ratio **b** = .0813 and FA ratio of .8023 are certainly reasonable numbers.

(Note that "best fit" is not claiming that a better fit by more fine tuning is not achievable; but this would almost certainly lead to slightly different values for different maturities.)

IV. Conclusion

Observed biases in option price data cannot be explained by the Black-Scholes model with its simple constant volatility parameter. Besides their fundamental call option pricing model Black and Scholes addressed issues that led inevitably to the volatility smile investigation. Attempts to eliminate the smile by modifying the underlying stock process, such as the CEV and stochastic volatility models mentioned above, have proven to be no more successful than Black-Scholes in resolving the smile paradox. The simplest conclusion from this is that the pricing of options on the equity of a firm cannot be explained by a single asset price process or distributional variations on that single process. Models with varying volatility introduce incomplete market solutions, without providing any economic insight into the cause for the volatility changes. Our solution offers an explanation for the observed variations in volatility based on the corporate balance sheet, as well as a non-constant equity return volatility produced by constant volatility processes.

Rubinstein's examination of the structure of the corporate balance sheet, followed the suggestion of Black and Scholes, but was limited due to concerns about the separate tradability of the underlying assets, in order to guarantee a risk-neutral valuation. Our formulation demonstrates that this concern was misplaced, and that the hedging of the sources of risk by distinct options was sufficient to guarantee a risk-neutral structure; the resulting option price was governed by a unique partial differential equation. In order to develop a closed form solution, we assumed that the net working capital and risky debt processes had a single underlying process which was independent of the risky asset process. This led to an equation resembling that obtained either from separately tradable assets or from the introduction of the market price of risk. The result is a tractable formula involving simple numerical integration.

Our example of a single trading event with four option maturities demonstrates that a consistent set of parameters can produce a set of option values that are much closer to the traded prices than does the Black-Scholes. Naturally, the flexibility of more parameter choice has to produce a better fit than the single implied volatility, but this preliminary result is very encouraging to further research and testing. This empirical demonstration offers support to our claim that the model is a better description of the true stock return dynamics.

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Endnotes

[1] The formulation for non-risky debt is explored in Ryan and Guo (2002); the result is a solution essentially the same as formulas (23) and (24) deriving from the PDE (17).

[2] See Rubinstein (1985) for tests of the CEV and Geske models.

Appendix

Proof of Lemma: Using Ito's formula, the instantaneous change of the value of the portfolio of calls C^A and C^B, given by expression (8), $P = \alpha C^A + \beta C^B + \gamma C^G - S$, is expressed as:

$$
\begin{aligned}
dP = \alpha\,[&C_t^A + \mu_1 U C_U^A + \mu_2 V C_V^A + \mu_3 L C_L^A + \tfrac{1}{2}(\sigma_1^2 U^2 C_{UU}^A + \sigma_2^2 V^2 C_{VV}^A + \sigma_3^2 L^2 C_{LL}^A) \\
&+ (\rho_{12}\sigma_1\sigma_2 UV C_{UV}^A + \rho_{13}\sigma_1\sigma_3 UL C_{UL}^A + \rho_{23}\sigma_2\sigma_3 VL C_{VL}^A)]dt \\
+ \beta\,[&C_t^B + \mu_1 U C_U^B + \mu_2 V C_V^B + \mu_3 L C_L^B + \tfrac{1}{2}(\sigma_1^2 U^2 C_{UU}^B + \sigma_2^2 V^2 C_{VV}^B + \sigma_3^2 L^2 C_{LL}^B) \\
&+ (\rho_{12}\sigma_1\sigma_2 UV C_{UV}^B + \rho_{13}\sigma_1\sigma_3 UL C_{UL}^B + \rho_{23}\sigma_2\sigma_3 VL C_{VL}^B)]\,dt \\
+ \gamma\,[&C_t^G + \mu_1 U C_U^G + \mu_2 V C_V^G + \mu_3 L C_L^G + \tfrac{1}{2}(\sigma_1^2 U^2 C_{UU}^G + \sigma_2^2 V^2 C_{VV}^G + \sigma_3^2 L^2 C_{LL}^G) \\
&+ (\rho_{12}\sigma_1\sigma_2 UV C_{UV}^G + \rho_{13}\sigma_1\sigma_3 UL C_{UL}^G + \rho_{23}\sigma_2\sigma_3 VL C_{VL}^G)]\,dt \\
&- \left(\mu_1 U + \mu_2 V - \mu_3 L\right)dt \\
&+ \left(\alpha\sigma_1 U C_U^A + \beta\sigma_1 U C_U^B + \gamma\sigma_1 U C_U^G - \sigma_1 U\right)dz_1 \\
&+ \left(\alpha\sigma_2 V C_V^A + \beta\sigma_2 V C_V^B + \gamma\sigma_1 V C_V^G - \sigma_2 V\right)dz_2 \\
&+ \left(\alpha\sigma_3 L C_L^A + \beta\sigma_3 L C_L^B + \gamma\sigma_3 L C_L^G + \sigma_3 L\right)dz_3,
\end{aligned}
\tag{A1}
$$

where C_U^A is the partial derivative of call A with respect to U (and similarly for the others), and ρ_{ij} is the correlation coefficient between processes i and j. The portfolio is riskless, if α, β and γ are chosen such that

$$\alpha C_U^A + \beta C_U^B + \gamma C_U^G = 1 \tag{A2}$$

$$\alpha C_V^A + \beta C_V^B + \gamma C_V^G = 1 \tag{A3}$$

$$\alpha C_L^A + \beta C_L^B + \gamma C_L^G = 1 \tag{A4}$$

as the coefficients of the last three terms in (A1) vanish; it is also independent of the drifts of both processes, as the terms involving μ_1, μ_2 and μ_3 vanish in the first four terms of (A1). By the Black-Scholes argument, the portfolio must then earn a riskless return $dP/dt = rP$; hence,

$$\begin{aligned}
&\{\tfrac{1}{2}\left(\sigma_1^2U^2C_{UU}^A + \sigma_2^2V^2C_{VV}^A + \sigma_3^2L^2C_{LL}^A\right) + \left(\rho_{12}\sigma_1\sigma_2UVC_{UV}^A + \rho_{13}\sigma_1\sigma_3ULC_{UL}^A + \rho_{23}\sigma_2\sigma_3VLC_{VL}^A\right) + C_t^A\}\alpha \\
&+ \{\tfrac{1}{2}\left(\sigma_1^2U^2C_{UU}^B + \sigma_2^2V^2C_{VV}^B + \sigma_3^2L^2C_{LL}^B\right) + \left(\rho_{12}\sigma_1\sigma_2UVC_{UV}^B + \rho_{13}\sigma_1\sigma_3ULC_{UL}^B + \rho_{23}\sigma_2\sigma_3VLC_{VL}^B\right) + C_t^B\}\beta \\
&+ \tfrac{1}{2}\left(\sigma_1^2U^2C_{UU}^G + \sigma_2^2V^2C_{VV}^G + \sigma_3^2L^2C_{LL}^G\right) + \left(\rho_{12}\sigma_1\sigma_2UVC_{UV}^G + \rho_{13}\sigma_1\sigma_3ULC_{UL}^G + \rho_{23}\sigma_2\sigma_3VLC_{VL}^G\right) + C_t^G\}\gamma \\
&= r\left(\alpha C^A + \beta C^B + \gamma C^G - S\right).
\end{aligned} \tag{A5}$$

Substituting *F(A), F(B)* and *F(G)* from equation (13) into equation (A5), we get:

$$\alpha F(A) + \beta F(B) + \gamma F(G) = -rS = -r(U + V - L), \tag{A6}$$

where the second equality is by the balance sheet equation (1); equations (A2),(A3),(A4) and (A6) are then the requisite system to guarantee a riskless portfolio.

Proof of Theorem:

In order for the system (8,9,10) or (A2,A3,A4,A5) of equations in three variables to be consistent, so that a riskless hedge portfolio (α, β, γ) can be formed, there must exist four non-trivial scalars $(\eta_1, \eta_2, \eta_3, \eta_4)$ such that the linear combination of the equations is zero.

That is, $[M[\alpha, \beta, \gamma, 1]^T = Z$ has a solution iff $[[\eta_1, \eta_2, \eta_3, \eta_4]M = Z^T$, for

$$M = \begin{bmatrix} C_U^A & C_U^B & C_U^G & -1 \\ C_V^A & C_V^B & C_V^G & -1 \\ C_L^A & C_L^B & C_L^G & -1 \\ F(A) & F(B) & F(G) & r(U + V - L) \end{bmatrix} \quad and \quad Z = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{A7}$$

As we are free to scale the scalars in a homogeneous equation, we choose η_4=1 to get:

$$\begin{aligned}
\eta_1C_U^A + \eta_2C_V^A + \eta_3C_L^A + F(A) = 0 \\
\eta_1C_U^B + \eta_2C_V^B + \eta_3C_L^B + F(B) = 0 \\
\eta_1C_U^G + \eta_2C_V^G + \eta_3C_L^G + F(G) = 0 \\
\eta_1 + \eta_2 + \eta_3 - r(U + V - L) = 0
\end{aligned} \tag{A8}$$

Since the first three equations are representative of any options chosen to hedge, we can solve for any one of them with the fourth and drop the superscripts; replacing η_1 by $-\eta_2 + \eta_3 + r(U + V - L)$ in the first equation and substituting for F(A), we have upon re-ordering terms:

$$
\begin{aligned}
&[C_t + rUC_U + rVC_V + rLC_L + \tfrac{1}{2}(\sigma_1^2 U^2 C_{UU} + \sigma_2^2 V^2 C_{VV} + \sigma_3^2 L^2 C_{LL}) \\
&+ (\rho_{12}\sigma_1\sigma_2 UVC_{UV} + \rho_{13}\sigma_1\sigma_3 ULC_{UL} + \rho_{23}\sigma_2\sigma_3 VLC_{VL}) - rC] \\
&+ (rV - \eta_2)(C_U - C_V) - (rL - \eta_3)(C_U + C_L) = 0
\end{aligned}
$$

The scalars η_2 and η_3 which are applied to (A3) and (A4) cannot depend on any option-specific parameters, such as the strike price and the time to expiration; hence their values must be $\eta_2 = rV$ and $\eta_3 = rL$, and the option-related terms in the first brackets must sum to zero. This gives us the risk-neutral or preference-free partial differential equation (14):

$$
\begin{aligned}
&C_t + rUC_U + rVC_V + rLC_L + \tfrac{1}{2}(\sigma_1^2 U^2 C_{UU} + \sigma_2^2 V^2 C_{VV} + \sigma_3^2 L^2 C_{LL}) \\
&+ (\rho_{12}\sigma_1\sigma_2 UVC_{UV} + \rho_{13}\sigma_1\sigma_3 ULC_{UL} + \rho_{23}\sigma_2\sigma_3 VLC_{VL}) - rC = 0
\end{aligned}
$$

[3] In general, for the a values used, the results were relatively insensitive to Φ_2.

Table 1. Call price values resulting from modifying a and b.

The table gives the values of expression (19) for the call value on the S&P 500 index on January 4, 1993 for an exercise price of X=425, index level of S = 436.96, time to expiry of t = 256 days, and riskless rate of r = 0.034. The B-S implied volatility of $\sigma = 0.148$ corresponding to an observed call price of 27.904, and assuming a continuous dividend yield of 3.5% and low-risk asset $\sigma_2 = 0.035$, is maintained as debt ratio b and fixed asset ratio a are varied, by adjusting high-risk σ_1 as shown in the first column. The call value rises monotonically in b for all a; for all b, the call value is unimodal in a, with a maximum at a = 0.35

σ_1 (for U)	F.A. (a)	b=.05	b=.15	b=.25	b=.35	b=.45	b=.05	b=.65	b=.75	b=.85	b=.95
2.8843	0.05	19.0854	20.2021	21.5637	23.1463	24.9103	26.8061	28.7812	30.7865	32.7821	34.7397
0.9665	0.15	30.8832	32.6045	34.3774	36.1900	38.0330	39.8998	41.7855	43.6864	45.5997	47.5233
0.5826	0.25	31.9936	33.7783	35.6028	37.4582	39.3377	41.2365	43.1509	45.0781	47.0159	48.9626
0.4178	0.35	32.2111	34.0034	35.8320	37.6887	39.5677	41.4646	43.3760	45.2993	47.2327	49.1744
0.3261	0.45	32.1466	33.9333	35.7546	37.6028	39.4724	41.3591	43.2599	45.1722	47.0942	49.0243
0.2676	0.55	31.9603	33.7364	35.5462	37.3823	39.2393	41.1130	43.0005	44.8992	0	0
0.2269	0.65	31.7113	33.4746	35.2712	37.0936	38.9366	0	0	0	0	0
0.1970	0.75	31.427	33.176	34.9586	0	0	0	0	0	0	0
0.1740	0.85	31.120	32.850	0	0	0	0	0	0	0	0
0.1558	0.95	0	0	0	0	0	0	0	0	0	0

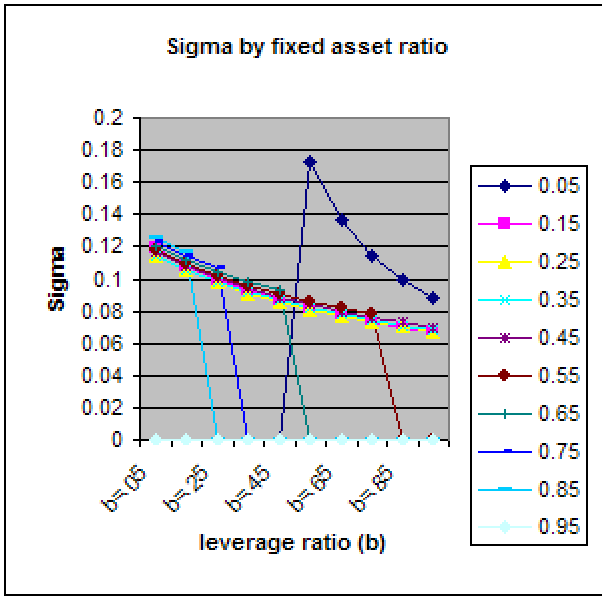
Table 2. Risk parameters corresponding to a fixed call price..

The table gives the values of the standard deviations for the call value on the S&P 500 index on January 4, 1993 for an exercise price of X=425, index level of S = 436.96, time to expiry of t = 256 days, and riskless rate of r = 0.034. The observed call price of 27.904, assuming a continuous dividend yield of 3% and low-risk asset $\sigma_2 = 0.035$, is maintained as debt ratio b and fixed asset ratio a are varied, by adjusting the value of total σ in panel A; the corresponding changes to high-risk σ_1 are shown in panel B. Both values are convex in both a and b and decrease monotonically in a for all b, and are U-shaped in b for all a. The B-S implied volatility of $\sigma = 0.148$ corresponds to the call price.

Panel A: Call price with X = 400										
F.A.=a	b=.05	b=.15	b=.25	b=.35	b=.45	b=.05	b=.65	b=.75	b=.85	b=.95
0.05	0	0	0	0	0	37.5605	36.8099	36.1114	35.5538	35.2240
0.15	45.3812	46.2079	46.4130	46.4535	46.4637	46.4663	46.4644	46.4586	46.4496	46.4377
0.25	46.4586	46.4420	46.4183	46.3893	46.3560	46.3194	46.2802	46.2390	46.1961	46.1520
0.35	46.3489	46.2962	46.2390	46.1787	46.1160	46.0517	45.9863	45.9202	45.8536	45.7868
0.45	46.1787	46.0978	46.0145	45.9297	45.8442	45.7582	45.6722	45.5867	45.5015	45.4171
0.55	45.9863	45.8822	45.7773	45.6723	45.5676	45.4639	45.3611	45.2594	0	0
0.65	45.7869	45.6628	45.5393	45.4170	45.2963	0	0	0	0	0
0.75	45.5867	45.4450	45.3054	0	0	0	0	0	0	0
0.85	45.3890	45.2418	0	0	0	0	0	0	0	0
0.95	0	0	0	0	0	0	0	0	0	0

Panel B: Call price with X = 450										
F.A.=a	b=.05	b=.15	b=.25	b=.35	b=.45	b=.05	b=.65	b=.75	b=.85	b=.95
0.05	0	0	0	0	0	28.0333	25.7291	23.7763	22.1590	20.8527
0.15	16.5047	16.2595	16.0678	15.9143	15.7902	15.6883	15.6044	15.5348	15.4769	15.4285
0.25	15.5348	15.4436	15.3761	15.3265	15.2903	15.2648	15.2478	15.2374	15.2326	15.2323
0.35	15.2844	15.2536	15.2375	15.2322	15.2349	15.2440	15.2582	15.2763	15.2975	15.3212
0.45	15.2321	15.2369	15.2516	15.2735	15.3008	15.3320	15.3662	15.4029	15.4411	15.4807
0.55	15.2581	15.2880	15.3247	15.3662	15.4110	15.4585	15.5076	15.5578	0	0
0.65	15.3213	15.3703	15.4239	15.4806	15.5395	0	0	0	0	0
0.75	15.4027	15.4672	15.5347	0	0	0	0	0	0	0
0.85	15.4940	15.5811	0	0	0	0	0	0	0	0
0.95	0	0	0	0	0	0	0	0	0	0

Table 2 - Panel B



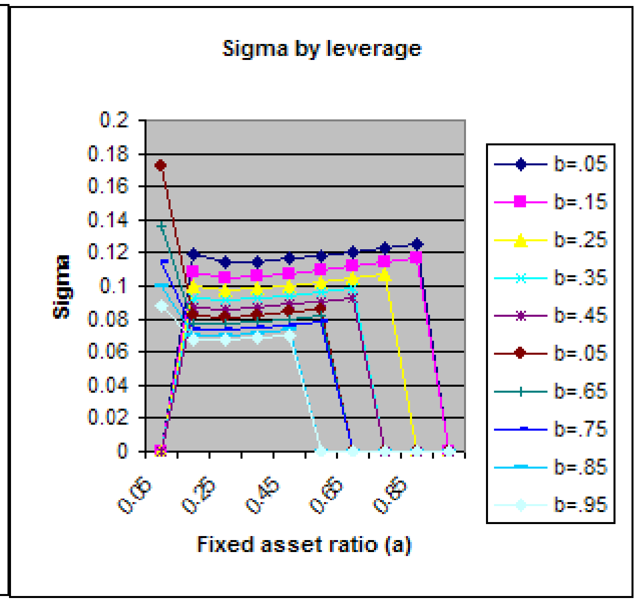
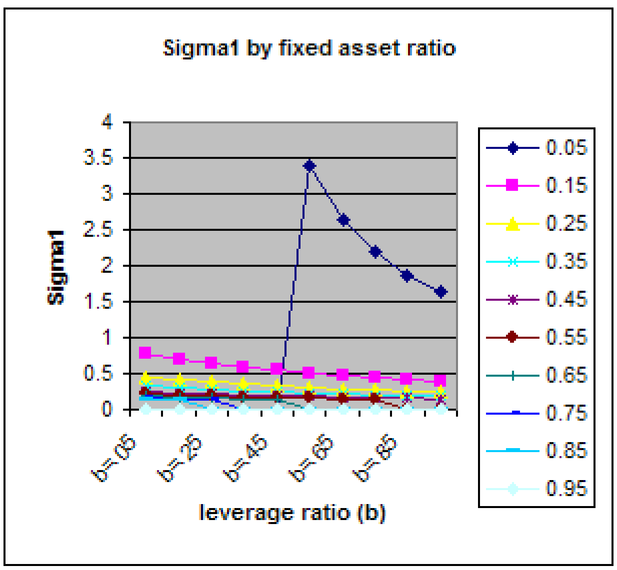
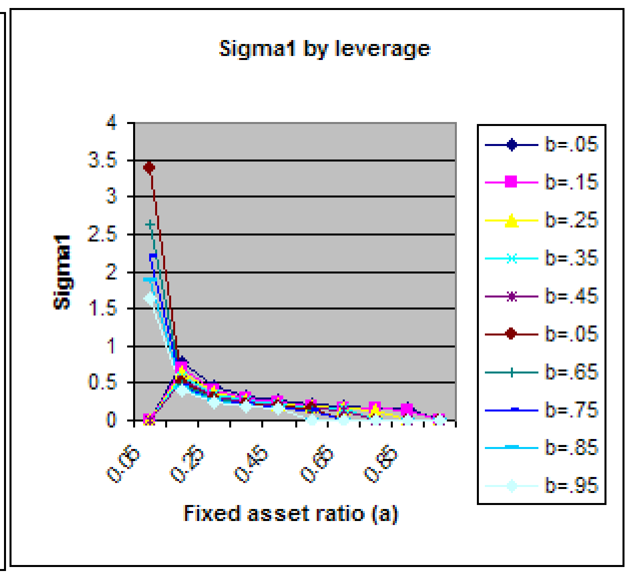


Table 2 - Panel B





(These figures are inserted to help interpret the data in Table 2.)

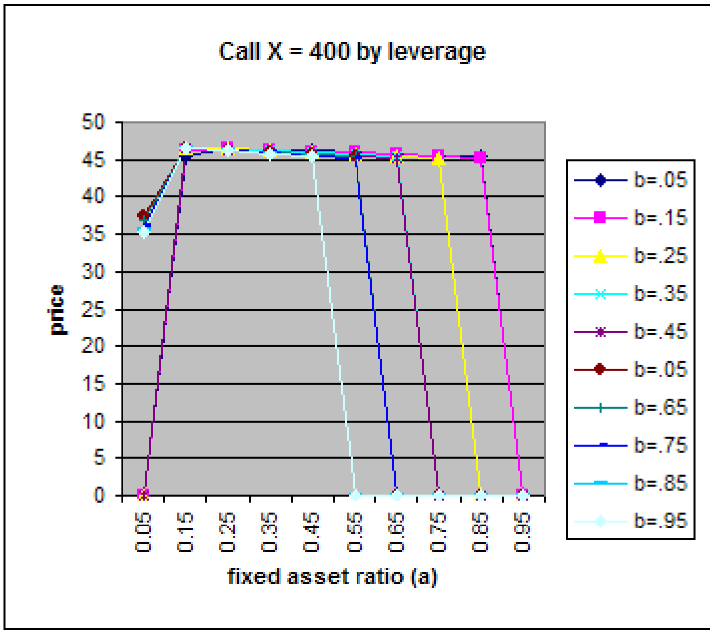
Table 3. Corresponding call prices for alternative exercise prices.

The table gives the values of the adjacent exercise price calls given a fixed call value on the S&P 500 index on January 4, 1993 for an exercise price of X=425, index level of S = 436.96, time to expiry of t = 256 days, and riskless rate of r = 0.034. The observed call price of 27.904, assuming a continuous dividend yield of 3% and low-risk asset σ_2 = 0.035, is maintained as debt ratio b and low-risk asset ratio 1-a are varied by adjusting the value of total σ as in Table 2. In panels A and B, respectively, the corresponding changes to calls with exercise prices X = 400 and X = 450 are given. The 400 call price is seen to be decreasing in **b** for all **a**; the price is concave in **a** for all **b.** In panel B, the 450 call values are convex in **a** for all **b**, and appear to be convex in **b** for all **a**.

Panel A: Call price with X = 400										
	b=.05	b=.15	b=.25	b=.35	b=.45	b=.05	b=.65	b=.75	b=.85	b=.95
0.05	0	0	0	0	0	37.5605	36.8099	36.1114	35.5538	35.2240
0.15	45.3812	46.2079	46.4130	46.4535	46.4637	46.4663	46.4644	46.4586	46.4496	46.4377
0.25	46.4586	46.4420	46.4183	46.3893	46.3560	46.3194	46.2802	46.2390	46.1961	46.1520
0.35	46.3489	46.2962	46.2390	46.1787	46.1160	46.0517	45.9863	45.9202	45.8536	45.7868
0.45	46.1787	46.0978	46.0145	45.9297	45.8442	45.7582	45.6722	45.5867	45.5015	45.4171
0.55	45.9863	45.8822	45.7773	45.6723	45.5676	45.4639	45.3611	45.2594	0	0
0.65	45.7869	45.6628	45.5393	45.4170	45.2963	0	0	0	0	0
0.75	45.5867	45.4450	45.3054	0	0	0	0	0	0	0
0.85	45.3890	45.2418	0	0	0	0	0	0	0	0
0.95	0	0	0	0	0	0	0	0	0	0

Panel B: Call price with X = 450										
	b=.05	b=.15	b=.25	b=.35	b=.45	b=.05	b=.65	b=.75	b=.85	b=.95
0.05	0	0	0	0	0	0	0	0	0	0
0.15	16.5047	16.2595	16.0678	15.9143	15.7902	15.6883	15.6044	15.5348	15.4769	15.4285
0.25	15.5348	15.4436	15.3761	15.3265	15.2903	15.2648	15.2478	15.2374	15.2326	15.2323
0.35	15.2844	15.2536	15.2375	15.2322	15.2349	15.2440	15.2582	15.2763	15.2975	15.3212
0.45	15.2321	15.2369	15.2516	15.2735	15.3008	15.3320	15.3662	15.4029	15.4411	15.4807
0.55	15.2581	15.2880	15.3247	15.3662	15.4110	15.4585	15.5076	15.5578	0	0
0.65	15.3213	15.3703	15.4239	15.4806	15.5395	0	0	0	0	0
0.75	15.4027	15.4672	15.5347	0	0	0	0	0	0	0
0.85	15.4940	15.5811	0	0	0	0	0	0	0	0
0.95	0	0	0	0	0	0	0	0	0	0

Table 3 - Panel A



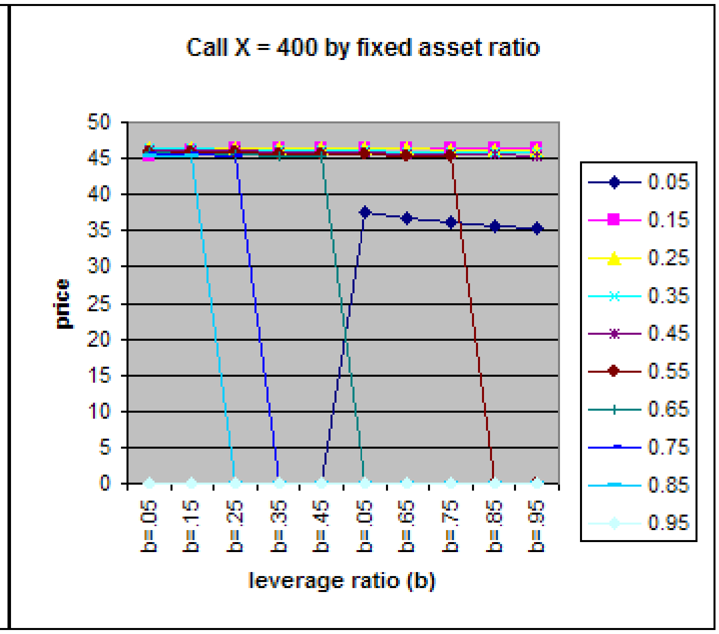
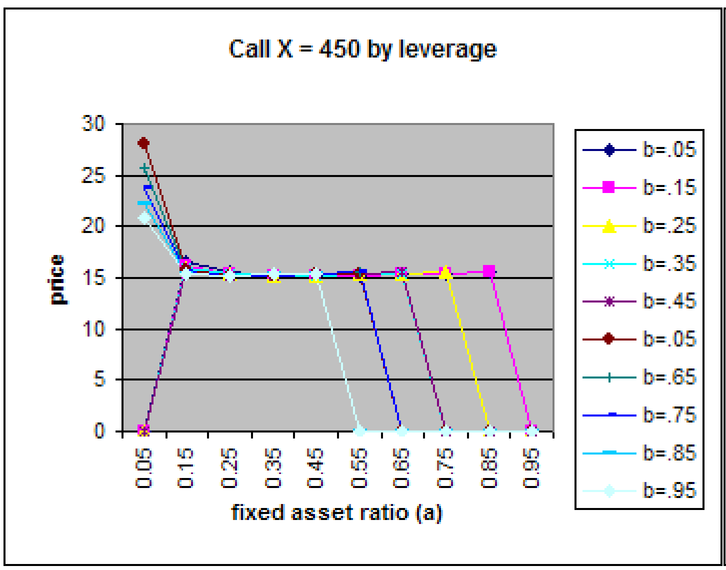
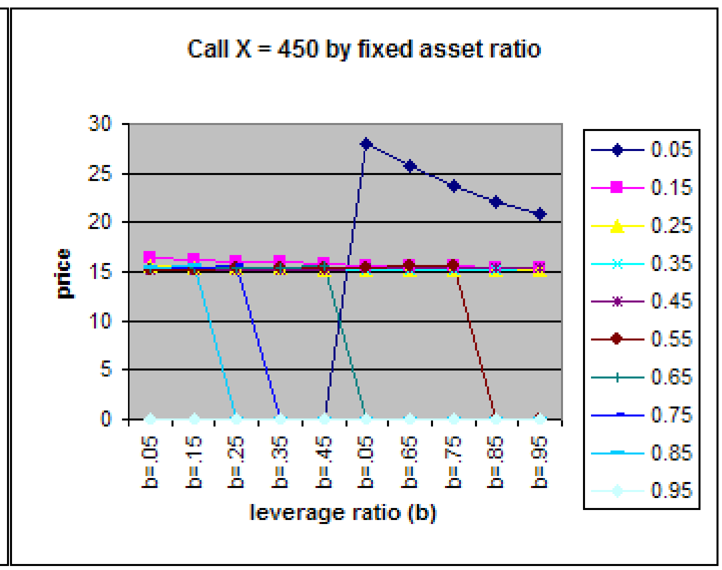


Table 3 - Panel B





(These figures are inserted to help interpret the data in Table 3.)

Figure 1 Pricing error for 1-year S&P 500 index options using the Black-Scholes and balance sheet models. The pricing error is defined as the difference between modeled price and market bid-asked mid-point for both models; sigma is .1054 compared to the B-S implied volatility of .1205 (for X= 425), and the market prices range from \$69 to \$19. Vertical order: {D/E = .0813, FA/TA = .8023, sigma = .1054}, Black-Scholes. Inputs: t = .9507, q = .01, r = .035, σ_2 =.035.

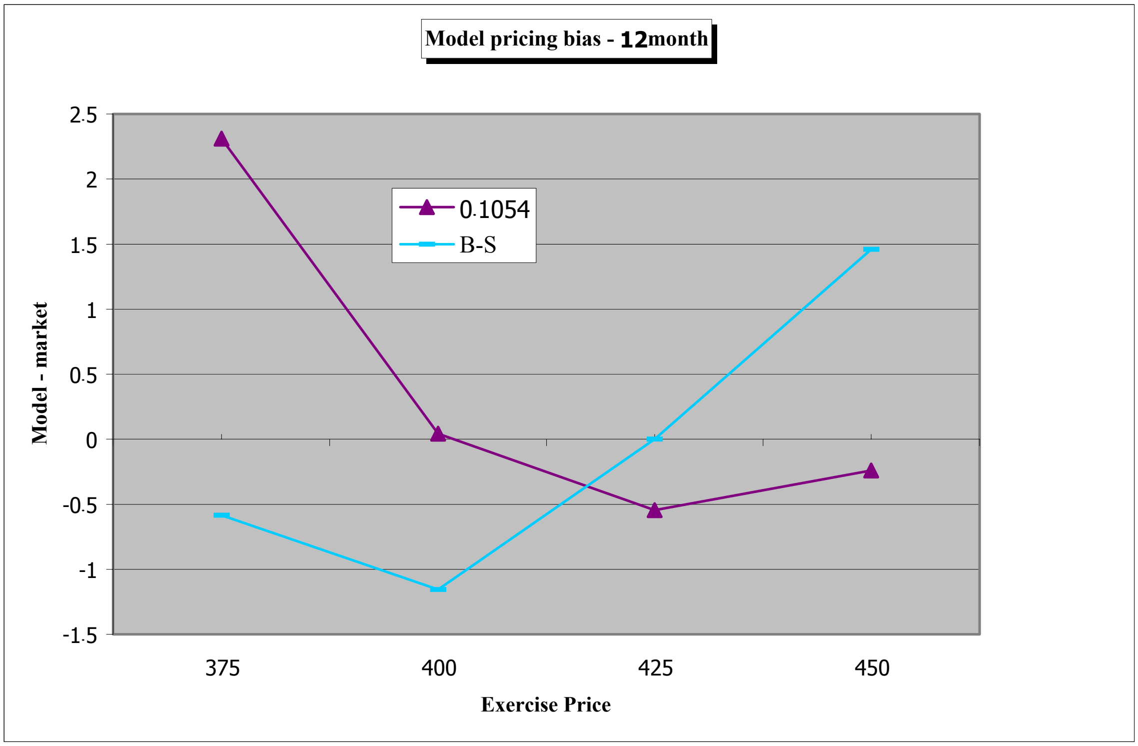


Figure 2 Pricing error for 9-month S&P 500 index options using the Black-Scholes and balance sheet models. The pricing error is defined as the difference between modeled price and market bid-asked mid-point for both models; sigma is .1054 compared to the B-S implied volatility of .1206 (for X= 425), and the market prices range from \$45 to \$15. Vertical order: {D/E = .0813, FA/TA = .8023, sigma = .1054}, Black-Scholes. Inputs: t = .7014, q = .01, r = .034, σ_2 =.035.

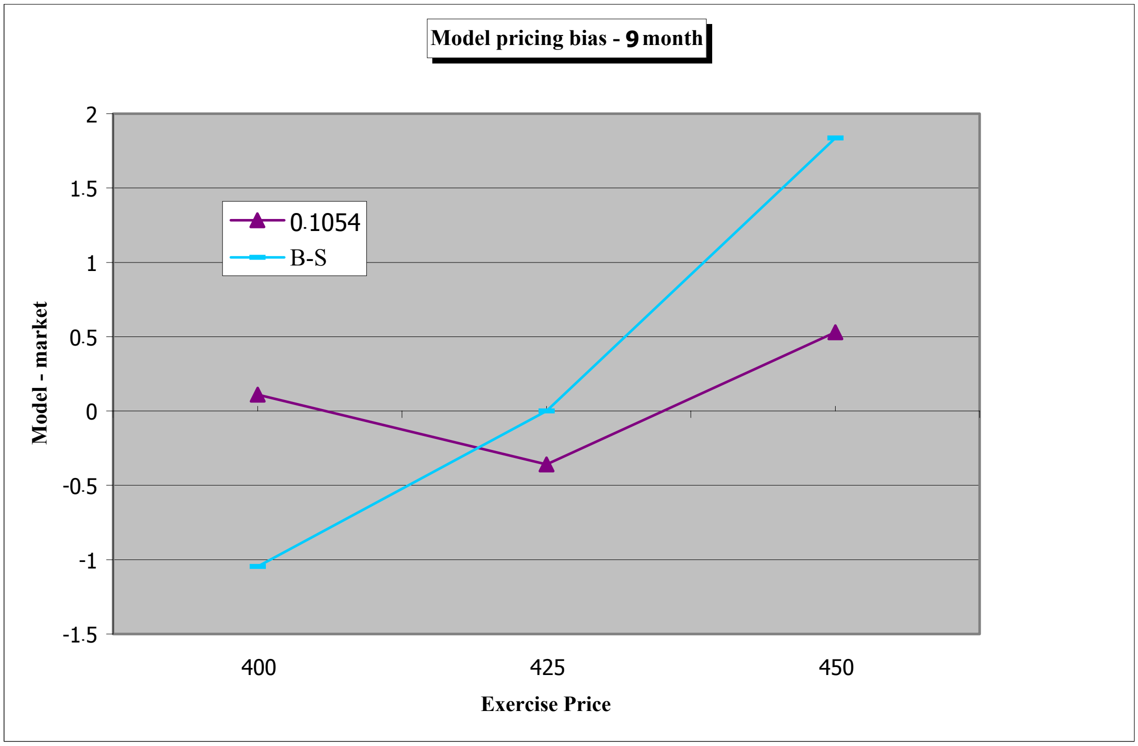


Figure 3 Pricing error for 6-month S&P 500 index options using the Black-Scholes and balance sheet models. The pricing error is defined as the difference between modeled price and market bid-asked mid-point for both models; sigma is .1054 compared to the B-S implied volatility of .1247 (for X= 435), and the market prices range from \$63 to \$6. Vertical order: {D/E = .0813, FA/TA = .8023, sigma = .1054}, Black-Scholes. Inputs: t = .4521, q = .01, r = .033, σ_2 =.035.

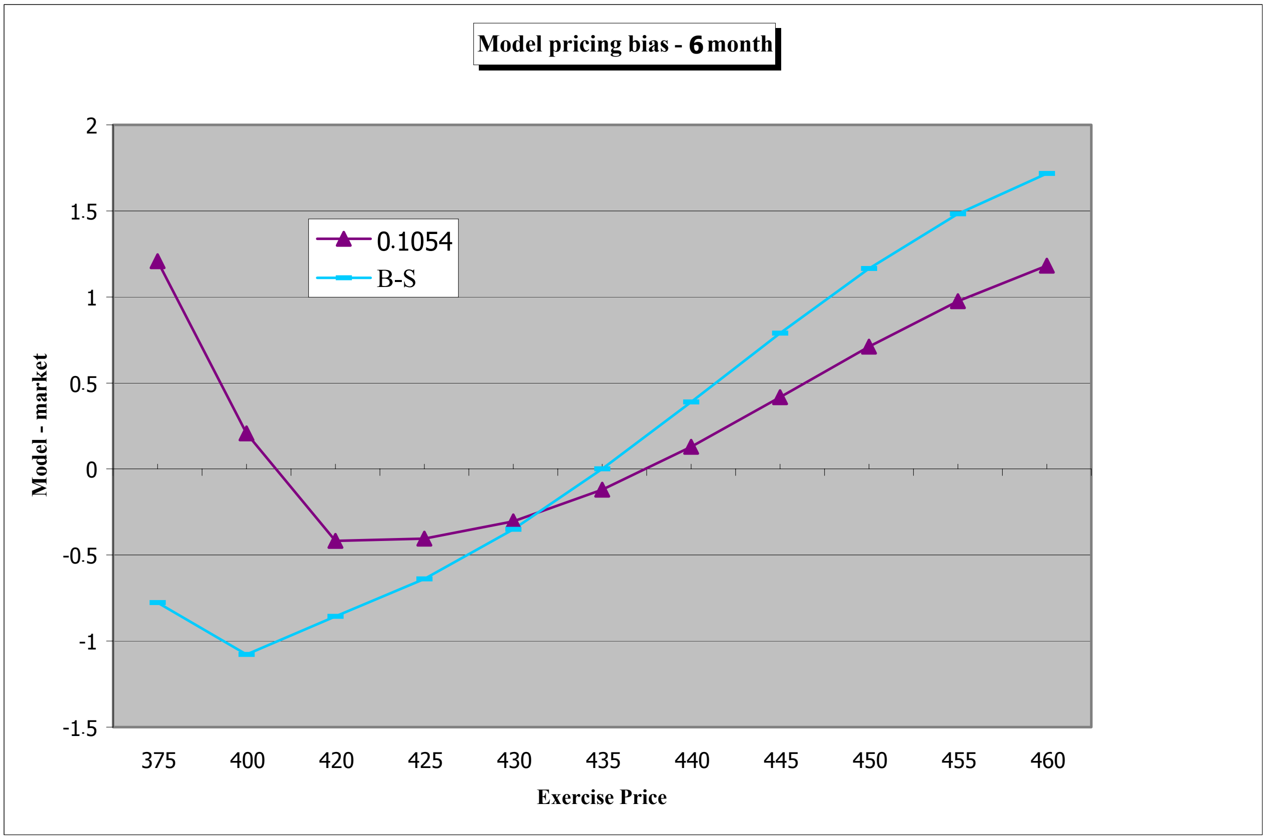


Figure 4 Pricing error for 3-month S&P 500 index options using the Black-Scholes and balance sheet models. The pricing error is defined as the difference between modeled price and market bid-asked mid-point for both models; sigma is .1054 compared to the B-S implied volatility of .1274 (for X= 435), and the market prices range from $62 to $2. Vertical order: {D/E = .0813, FA/TA = .8023, sigma = .1054}, Black-Scholes. Inputs: t = .2027, q = .01, r = .032, σ_2 =.035.